

# CONTENTS

<i>Preface to the Third Edition</i>	ix
<b>1. Introduction</b>	<b>1</b>
1.1. Classification of Parameter Estimation and Inverse Problems	1
1.2. Examples of Parameter Estimation Problems	4
1.3. Examples of Inverse Problems	8
1.4. Discretizing Integral Equations	13
1.5. Why Inverse Problems Are Hard	18
1.6. Exercises	21
1.7. Notes and Further Reading	22
<b>2. Linear Regression</b>	<b>25</b>
2.1. Introduction to Linear Regression	25
2.2. Statistical Aspects of Least Squares	27
2.3. An Alternative View of the 95% Confidence Ellipsoid	37
2.4. Unknown Measurement Standard Deviations	39
2.5. $L_1$ Regression	43
2.6. Monte Carlo Error Propagation	48
2.7. Exercises	49
2.8. Notes and Further Reading	53
<b>3. Rank Deficiency and Ill-Conditioning</b>	<b>55</b>
3.1. The SVD and the Generalized Inverse	55
3.2. Covariance and Resolution of the Generalized Inverse Solution	61
3.3. Instability of the Generalized Inverse Solution	64
3.4. A Rank Deficient Tomography Problem	67
3.5. Discrete Ill-Posed Problems	74
3.6. Exercises	88
3.7. Notes and Further Reading	91
<b>4. Tikhonov Regularization</b>	<b>93</b>
4.1. Selecting a Good Solution	93
4.2. SVD Implementation of Tikhonov Regularization	95
4.3. Resolution, Bias, and Uncertainty in the Tikhonov Solution	100
4.4. Higher-Order Tikhonov Regularization	103
4.5. Resolution in Higher-Order Tikhonov Regularization	111
4.6. The TGSVD Method	113
4.7. Generalized Cross-Validation	116
4.8. Error Bounds	120
4.9. Using Bounds as Constraints	125
4.10. Exercises	130

4.11. Notes and Further Reading	133
<b>5. Discretizing Inverse Problems Using Basis Functions</b>	<b>135</b>
5.1. Discretization by Expansion of the Model	135
5.2. Using Representers as Basis Functions	140
5.3. Reformulation in Terms of an Orthonormal Basis	141
5.4. The Method of Backus and Gilbert	143
5.5. Exercises	147
5.6. Notes and Further Reading	148
<b>6. Iterative Methods</b>	<b>151</b>
6.1. Introduction	151
6.2. Row Action Methods for Tomography Problems	152
6.3. The Gradient Descent Method	156
6.4. The Conjugate Gradient Method	160
6.5. The CGLS Method	165
6.6. Resolution Analysis for Iterative Methods	170
6.7. Exercises	176
6.8. Notes and Further Reading	179
<b>7. Sparsity Regularization and Total Variation Techniques</b>	<b>181</b>
7.1. Sparsity Regularization	181
7.2. The Iterative Soft Thresholding Algorithm (ISTA)	182
7.3. Sparse Representation and Compressive Sensing	189
7.4. Total Variation Regularization	195
7.5. Using IRLS to Solve $L_1$ Regularized Problems	196
7.6. The Alternating Direction Method of Multipliers (ADMM)	198
7.7. Total Variation Image Denoising	205
7.8. Exercises	208
7.9. Notes and Further Reading	209
<b>8. Fourier Techniques</b>	<b>211</b>
8.1. Linear Systems in the Time and Frequency Domains	211
8.2. Linear Systems in Discrete Time	217
8.3. Water Level Regularization	221
8.4. Tikhonov Regularization in the Frequency Domain	225
8.5. Exercises	230
8.6. Notes and Further Reading	233
<b>9. Nonlinear Regression</b>	<b>235</b>
9.1. Introduction to Nonlinear Regression	235
9.2. Newton's Method for Solving Nonlinear Equations	235
9.3. The Gauss–Newton and Levenberg–Marquardt Methods for Solving Nonlinear Least Squares Problems	238
9.4. Statistical Aspects of Nonlinear Least Squares	241

9.5. Implementation Issues	246
9.6. Exercises	252
9.7. Notes and Further Reading	255
<b>10. Nonlinear Inverse Problems</b>	<b>257</b>
10.1. Regularizing Nonlinear Least Squares Problems	257
10.2. Occam's Inversion	262
10.3. Model Resolution in Nonlinear Inverse Problems	266
10.4. The Nonlinear Conjugate Gradient Method	269
10.5. The Discrete Adjoint Method	270
10.6. Exercises	276
10.7. Notes and Further Reading	277
<b>11. Bayesian Methods</b>	<b>279</b>
11.1. Review of the Classical Approach	279
11.2. The Bayesian Approach	281
11.3. The Multivariate Normal Case	286
11.4. The Markov Chain Monte Carlo (MCMC) Method	295
11.5. Analyzing MCMC Output	299
11.6. Exercises	303
11.7. Notes and Further Reading	305
<b>12. Epilogue</b>	<b>307</b>
<b>A. Review of Linear Algebra</b>	<b>309</b>
A.1. Systems of Linear Equations	309
A.2. Matrix and Vector Algebra	312
A.3. Linear Independence	318
A.4. Subspaces of $\mathbf{R}^n$	319
A.5. Orthogonality and the Dot Product	324
A.6. Eigenvalues and Eigenvectors	328
A.7. Vector and Matrix Norms	330
A.8. The Condition Number of a Linear System	332
A.9. The QR Factorization	334
A.10. Complex Matrices and Vectors	336
A.11. Linear Algebra in Spaces of Functions	337
A.12. Exercises	338
A.13. Notes and Further Reading	340
<b>B. Review of Probability and Statistics</b>	<b>341</b>
B.1. Probability and Random Variables	341
B.2. Expected Value and Variance	347
B.3. Joint Distributions	348
B.4. Conditional Probability	352
B.5. The Multivariate Normal Distribution	354

B.6.	The Central Limit Theorem	355
B.7.	Testing for Normality	356
B.8.	Estimating Means and Confidence Intervals	358
B.9.	Exercises	360
B.10.	Notes and Further Reading	361
<b>C.</b>	<b>Review of Vector Calculus</b>	<b>363</b>
C.1.	The Gradient, Hessian, and Jacobian	363
C.2.	Taylor's Theorem	364
C.3.	Lagrange Multipliers	365
C.4.	Exercises	368
C.5.	Notes and Further Reading	369
<b>D.</b>	<b>Glossary of Notation</b>	<b>371</b>
	<i>Bibliography</i>	373
	<i>Index</i>	383
<b>12. Epilogue</b>		
<b>A. Review of Linear Algebra</b>		
A.1.	Systems of Linear Equations	387
A.2.	Matrix and Vector Algebra	391
A.3.	Linear Independence	395
A.4.	Subspaces of $\mathbb{R}^n$	397
A.5.	Orthogonality and the Dot Product	401
A.6.	Eigenvalues and Eigenvectors	405
A.7.	Vector and Matrix Norms	411
A.8.	The Condition Number of a Matrix	415
A.9.	The QR Factorization	419
A.10.	Complex Matrices and Vectors	423
A.11.	Linear Algebra in Sobolev Spaces	427
A.12.	Exercises	431
A.13.	Notes and Further Reading	435
<b>B. Review of Probability and Statistics</b>		
B.1.	Probability and Random Variables	437
B.2.	Ordered Data and Rank Statistics	441
B.3.	Bayesian Inference	445
B.4.	Conditional Probability	449
B.5.	The Multivariate Normal Distribution	453