

Contents

Preface	v
A Brief Reader's Guide	xi
Chapter 1. Introductory Topics	1
1.1 Some General Notions	1
1.2 Motivating Examples	3
1.2.1 Cost Functionals	3
1.2.2 Partial Differential Equations Setting	6
1.2.3 Applications	14
1.2.4 Variable Domains	20
Chapter 2. Existence	25
2.1 A General Situation	25
2.2 Special Existence and Uniqueness Results	33
2.2.1 Second-Order Problems	33
2.2.2 Fourth-Order Problems	39
2.3 Variable Domains	44
2.3.1 Some Examples	45
2.3.2 General Dirichlet Problems	53
2.3.3 Neumann and Mixed Boundary Conditions	64
2.3.4 Partial Extensions	75
Chapter 3. Optimality Conditions	83
3.1 Abstract Approaches	83
3.1.1 The Convex Case. Subdifferential Calculus	83
3.1.2 The Convex Case. Mathematical Programming	88
3.1.3 The Differentiable Case	92
3.1.3.1 Singular Control Problems	96

3.2	Penalization	102
3.2.1	The Standard Approach	103
3.2.2	Penalization of the State Equation	113
3.2.3	Semilinear Equations and Exact Penalization	127
3.3	Control of Variational Inequalities	143
3.3.1	An Abstract Result	143
3.3.2	Semilinear Variational Inequalities	148
3.3.3	State Constraints and Penalization of the Equation	159
3.4	Thickness Optimization for Plates	168
3.4.1	Simply Supported Plates	168
3.4.2	Clamped Plates and Control Variational Methods	175
3.4.2.1	Penalization and Variational Inequalities	183
	Chapter 4. Discretization	193
4.1	Finite Element Approximation of Elliptic Equations	193
4.1.1	The Finite Element Method	194
4.1.2	Error Estimates for the FE Equations	198
4.2	Error Estimates in the Finite Element Discretization of Control Problems	206
4.3	Semidiscretization	220
4.4	Optimal Control Problems Governed by Elliptic Variational Inequalities	226
4.4.1	The Ritz–Galerkin Approximation	228
4.5	Error Estimates in the Discretization of Control Problems with Nonlinear State Equation	238
	Chapter 5. Unknown Domains	249
5.1	Free Boundary Problems	249
5.1.1	The Dam Problem	250
5.1.2	Free Boundary Problems and Optimal Design	254
5.1.3	Shape Optimization of Systems with Free Boundaries	258
5.1.4	Controllability of the Coincidence Set	267
5.2	Direct Approaches	278
5.2.1	An Algorithm of C�ea, Gioan, and Michel	278
5.2.2	Characteristic Functions	284
5.2.2.1	Structural Material Optimization	285

5.2.2.2	Bang-Bang Controls and Characteristic Functions	294
5.2.3	Controllability and Fictitious Domains Approaches	297
5.2.3.1	Boundary Controllability	297
5.2.3.2	Distributed Controls	310
5.3	Domain Variations	323
5.3.1	Classical Approaches	323
5.3.2	Topological Asymptotics	334
Chapter 6. Optimization of Curved Mechanical Systems		341
6.1	Kirchhoff-Love Arches	342
6.1.1	Application of the Control Variational Method	343
6.1.2	Optimization of Nonsmooth Arches	351
6.1.3	Variational Inequalities for Arches	369
6.2	General Three-Dimensional Curved Rods	371
6.2.1	A Local Frame Under Low Differentiability Assumptions	372
6.2.2	A Generalized Naghdi-Type Model	374
6.2.3	Shape Optimization	389
6.3	Applications to Shells	406
6.3.1	A Generalized Naghdi Shell Model	406
6.3.2	Proof of Coercivity	413
6.3.3	Shell Optimal Design	422
Appendix 1. Convex Mappings and Monotone Operators		437
Appendix 2. Elliptic Equations and Variational Inequalities		451
Appendix 3. Domain Convergence		459
Bibliography		475
Index		495
Notation		505