Contents

Preface 11
Notation 13
1. Introduction 19
1.1 Object and history of the dynamics of railway bridges 19
1.2 Deterministic vibration 22
1.3 Dynamic coefficient 24
1.4 Stochastic vibration 26

2. Theoretical bridge models 29
2.1 Beams 29
2.1.1 Mass beams 29
2.1.2 Massless beams and other special cases 30
2.1.3 Continuous beams 31
2.1.4 Boundary and initial conditions 32
2.2 Plates 33
2.3 Complex systems 34
2.3.1 Trusses 34
2.3.2 Frames 35
2.3.3 Curved bars 35
2.4 Lumped masses and models with continuously distributed mass 37
2.5 Bridge deck modelling 38
2.5.1 Cross-beam effect 38
2.5.2 Ballast 41
2.6 Modelling other factors 42
2.6.1 Variable cross section 42
2.6.2 Prestressed concrete bridges 43
2.6.3 Influence of elastic foundation, shear and rotatory inertia 43
2.7 Modelling of railway bridges 45

3. Modelling of railway vehicles 47
3.1 Moving vertical forces 48
3.1.1 Constant forces 48
3.1.2 Harmonic variable force 49
3.1.3 Continuous load 49
3.1.4 Random load 50
3.2 Mass elements 51
3.2.1 Lumped mass 52
3.2.2 Rigid plates and bodies 53
3.3 Springs and damping elements 54
3.3.1 Linear spring 55
3.3.2 Non-linear spring 55
3.3.3 Stop 56
3.3.4 Viscous damping 57

3.3.5 Friction 57
3.3.6 Elastic-plastic element 58
3.4 Modelling of a bridge and a running train 59
3.4.1 Initial assumptions 59
3.4.2 Equations of motion 61
3.4.3 Movement of the vehicle along the bridge 63

4. Natural frequencies of railway bridges 66
4.1 Calculation of natural frequencies 66
4.1.1 Beams 67
4.1.2 Continuous beams 73
4.1.3 Plates 77
4.1.4 Natural frequencies of loaded bridges 77
4.2 Experimental results 80
4.2.1 Dynamic bridge stiffness 80
4.2.2 Statistical evaluation of natural frequencies 85
4.2.3 Empirical formulae 90

5. Damping of railway bridges 94
5.1 Damped vibrations of a beam during the passage of a force 94
5.1.1 Viscous damping proportional to the velocity of vibration 95
5.1.2 Dry friction 99
5.1.3 Complex theory of internal damping 104
5.2 Experimental results 107
5.2.1 Statistical evaluation of logarithmic decrement of damping 107
5.2.2 Empirical formulae 110

6. Influence of vehicle speed on dynamic stresses of bridges 112
6.1 Constant velocity of motion 112
6.2 Variable velocity of motion 116

7. Influence of track irregularities and other parameters 120
7.1 Periodic irregularities 121
7.1.1 Impact of wheel flats 123
7.1.2 Cross-beam and sleeper effects 124
7.1.3 Isolated irregularities 126
7.2 Random irregularities 126
7.3 Further parameters 129

8. Horizontal longitudinal effects on bridges 130
8.1 Motion of a disc rolling along a beam taking into account adhesion 130
8.1.1 Solution 132
8.1.2 Influence of some parameters 136
8.2 Quasistatic model 139
8.2.1 Solution 141
8.2.2 Influence of some parameters 146
8.2.3 Experiments on bridges 156
8.3 Starting and braking forces on bridges 158

9. Horizontal transverse effects on bridges 162
9.1 Beam 162
9.1.1 Vertical vibration 163
9.1.2 Horizontal vibration 166

9.1.3 Torsional vibration 168
9.2 Thin-walled bar with vertical axis of symmetry 170
9.2.1 Approximate solution 171
9.2.2 Dynamic solution 173
9.2.3 Thin-walled bar with two axes of symmetry 175
9.3 Experiments on bridges 175
9.4 Coefficients of variation for horizontal and torsional vibrations 177
9.5 Centrifugal forces 181

10. Traffic loads on railway bridges 183
10.1 Axle forces 185
10.1.1 Measurements of axle forces 185
10.1.2 Vertical axle forces 189
10.1.3 Horizontal transverse wheel forces 195
10.2 Axle spacing 195
10.3 Velocities 197

11. Statistical counting methods for the classification of random stress-time history 199
11.1 Statistical counting methods 200
11.1.1 Sampling method 200
11.1.2 Threshold method 201
11.1.3 Peak counting methods 201
11.1.4 Level crossing methods 202
11.1.5 Stress range counting methods 203
11.1.6 Multiparametric methods 203
11.2 Rain-flow counting method 204
11.2.1 Justification of the counting method from the $\sigma(\varepsilon)$ diagram 205
11.2.2 Counting rules 207
11.2.3 Algorithm for a computer 208
11.3 Appreciation of counting methods 212
11.4 Statistical evaluation of stresses 213
11.4.1 Stress extremes during train passage 213
11.4.2 Statistical evaluation of results of the sampling method 215

12. Stress ranges in steel railway bridges 217
12.1 Theoretical calculation of stress spectra 217
12.1.1 Characteristic trains 217
12.1.2 Traffic loads 221
12.1.3 Bending moment spectra 222
12.2 Experimental stress spectra 229
12.2.1 Empirical formula for the number of stress ranges 231
12.2.2 Probability density of stress ranges 231
12.2.3 Number of stress cycles per year 235
12.2.4 Number of stress cycles 239
12.3 Growing traffic loads 240
12.4 Influence of overloading 241
12.5 Other factors 242
12.6 Appreciation of stress spectra 243

13. The assessment of steel railway bridges for fatigue 244
13.1 Theory of fatigue damage accumulation 244
13.2 Method of equivalent damage 247

13.3 Limit state for fatigue 250
13.4 Fatigue assessment of bridges according to limit states theory 251
13.5 Propagation of fatigue cracks 253
13.6 Fatigue life and interval of railway bridge inspections 258
13.6.1 Residual service life for initial crack a_0 258
13.6.2 Service life assuming fatigue damage accumulation 258
13.6.3 Interval of railway bridge inspections 260

Appendix 262

14. Thermal interaction of long-welded rail with railway bridges 262
14.1 Theoretical model of long-welded rail and bridge 262
14.1.1 Basic assumptions 262
14.1.2 Basic equations and their solution 263
14.1.3 Examples 273
14.2 Comparison of theory with experiments 275
14.3 Expansion length of bridges with long-welded rail 278
14.3.1 Maximum and minimum temperatures 278
14.3.2 Strength condition 280
14.3.3 Gap condition 282
14.3.4 Mutual displacement condition 285
14.3.5 Stability condition 288
14.3.6 Admissible expansion length of bridges 288
14.4 Horizontal forces in bridges due to temperature changes 291
14.5 Effect of some parameters 305
14.5.1 Rail displacement 305
14.5.2 Rail force 305
14.5.3 Mutual displacement 306
14.5.4 Force in fixed bearings 307
14.5.5 Uniform load subjecting the bridge with curved rail 307
14.5.6 Strength condition 307
14.5.7 Gap condition 307
14.5.8 Mutual displacement condition 309
14.6 Conclusions for the application of long-welded rail on bridges 309
Bibliography 311
Author index 322
Subject index 326