

---

# Contents

---

|   |           |
|---|-----------|
| Preface   | xv        |
| <b>1 Introduction</b>   | <b>1</b>  |
| 1.1 The Need for Spatial Analysis                                     | 1         |
| 1.2 Types of Spatial Data   | 6         |
| 1.2.1 Geostatistical Data   | 7         |
| 1.2.2 Lattice Data, Regional Data                                     | 8         |
| 1.2.3 Point Patterns  | 11        |
| 1.3 Autocorrelation—Concept and Elementary Measures                   | 14        |
| 1.3.1 Mantel's Tests for Clustering                                   | 14        |
| 1.3.2 Measures on Lattices  | 18        |
| 1.3.3 Localized Indicators of Spatial Autocorrelation                 | 23        |
| 1.4 Autocorrelation Functions   | 25        |
| 1.4.1 The Autocorrelation Function of a Time Series                   | 25        |
| 1.4.2 Autocorrelation Functions in Space—Covariance and Semivariogram | 26        |
| 1.4.3 From Mantel's Statistic to the Semivariogram                    | 29        |
| 1.5 The Effects of Autocorrelation on Statistical Inference           | 31        |
| 1.5.1 Effects on Prediction   | 32        |
| 1.5.2 Effects on Precision of Estimators                              | 34        |
| 1.6 Chapter Problems  | 37        |
| <b>2 Some Theory on Random Fields</b>                                 | <b>41</b> |
| 2.1 Stochastic Processes and Samples of Size One                      | 41        |
| 2.2 Stationarity, Isotropy, and Heterogeneity                         | 42        |
| 2.3 Spatial Continuity and Differentiability                          | 48        |
| 2.4 Random Fields in the Spatial Domain                               | 52        |
| 2.4.1 Model Representation  | 53        |
| 2.4.2 Convolution Representation                                      | 57        |
| 2.5 Random Fields in the Frequency Domain                             | 62        |
| 2.5.1 Spectral Representation of Deterministic Functions              | 62        |
| 2.5.2 Spectral Representation of Random Processes                     | 65        |
| 2.5.3 Covariance and Spectral Density Function                        | 66        |
| 2.5.4 Properties of Spectral Distribution Functions                   | 70        |
| 2.5.5 Continuous and Discrete Spectra                                 | 72        |
| 2.5.6 Linear Location-Invariant Filters                               | 74        |

|          |  |            |
|----------|--|------------|
| 2.5.7    | Importance of Spectral Analysis  | 77         |
| 2.6      | Chapter Problems   | 78         |
| <b>3</b> | <b>Mapped Point Patterns</b>   | <b>81</b>  |
| 3.1      | Random, Aggregated, and Regular Patterns                                 | 81         |
| 3.2      | Binomial and Poisson Processes   | 83         |
| 3.2.1    | Bernoulli and Binomial Processes   | 83         |
| 3.2.2    | Poisson Processes  | 84         |
| 3.2.3    | Process Equivalence  | 85         |
| 3.3      | Testing for Complete Spatial Randomness                                  | 86         |
| 3.3.1    | Monte Carlo Tests  | 87         |
| 3.3.2    | Simulation Envelopes   | 88         |
| 3.3.3    | Tests Based on Quadrat Counts  | 90         |
| 3.3.4    | Tests Based on Distances   | 97         |
| 3.4      | Second-Order Properties of Point Patterns                                | 99         |
| 3.4.1    | The Reduced Second Moment Measure—<br>The $K$ -Function                  | 101        |
| 3.4.2    | Estimation of $K$ - and $L$ -Functions                                   | 102        |
| 3.4.3    | Assessing the Relationship between Two Patterns                          | 103        |
| 3.5      | The Inhomogeneous Poisson Process  | 107        |
| 3.5.1    | Estimation of the Intensity Function                                     | 110        |
| 3.5.2    | Estimating the Ratio of Intensity Functions                              | 112        |
| 3.5.3    | Clustering and Cluster Detection   | 114        |
| 3.6      | Marked and Multivariate Point Patterns                                   | 118        |
| 3.6.1    | Extensions   | 118        |
| 3.6.2    | Intensities and Moment Measures for Multivariate<br>Point Patterns       | 120        |
| 3.7      | Point Process Models   | 122        |
| 3.7.1    | Thinning and Clustering  | 123        |
| 3.7.2    | Clustered Processes  | 125        |
| 3.7.3    | Regular Processes  | 128        |
| 3.8      | Chapter Problems   | 129        |
| <b>4</b> | <b>Semivariogram and Covariance Function Analysis<br/>and Estimation</b> | <b>133</b> |
| 4.1      | Introduction   | 133        |
| 4.2      | Semivariogram and Covariogram  | 135        |
| 4.2.1    | Definition and Empirical Counterparts                                    | 135        |
| 4.2.2    | Interpretation as Structural Tools                                       | 138        |
| 4.3      | Covariance and Semivariogram Models                                      | 141        |
| 4.3.1    | Model Validity   | 141        |
| 4.3.2    | The Matérn Class of Covariance Functions                                 | 143        |
| 4.3.3    | The Spherical Family of Covariance Functions                             | 145        |
| 4.3.4    | Isotropic Models Allowing Negative Correlations                          | 146        |
| 4.3.5    | Basic Models Not Second-Order Stationary                                 | 149        |
| 4.3.6    | Models with Nugget Effects and Nested Models                             | 150        |

|          |   |            |
|----------|---|------------|
| 4.3.7    | Accommodating Anisotropy                                  | 151        |
| 4.4      | Estimating the Semivariogram                              | 153        |
| 4.4.1    | Matheron's Estimator                                      | 153        |
| 4.4.2    | The Cressie-Hawkins Robust Estimator                      | 159        |
| 4.4.3    | Estimators Based on Order Statistics and Quantiles        | 161        |
| 4.5      | Parametric Modeling                                       | 163        |
| 4.5.1    | Least Squares and the Semivariogram                       | 164        |
| 4.5.2    | Maximum and Restricted Maximum Likelihood                 | 166        |
| 4.5.3    | Composite Likelihood and Generalized Estimating Equations | 169        |
| 4.5.4    | Comparisons   | 172        |
| 4.6      | Nonparametric Estimation and Modeling                     | 178        |
| 4.6.1    | The Spectral Approach                                     | 179        |
| 4.6.2    | The Moving-Average Approach                               | 183        |
| 4.6.3    | Incorporating a Nugget Effect                             | 186        |
| 4.7      | Estimation and Inference in the Frequency Domain          | 188        |
| 4.7.1    | The Periodogram on a Rectangular Lattice                  | 190        |
| 4.7.2    | Spectral Density Functions                                | 198        |
| 4.7.3    | Analysis of Point Patterns                                | 200        |
| 4.8      | On the Use of Non-Euclidean Distances in Geostatistics    | 204        |
| 4.8.1    | Distance Metrics and Isotropic Covariance Functions       | 205        |
| 4.8.2    | Multidimensional Scaling                                  | 206        |
| 4.9      | Supplement: Bessel Functions                              | 210        |
| 4.9.1    | Bessel Function of the First Kind                         | 210        |
| 4.9.2    | Modified Bessel Functions of the First and Second Kind    | 210        |
| 4.10     | Chapter Problems  | 211        |
| <b>5</b> | <b>Spatial Prediction and Kriging</b>                     | <b>215</b> |
| 5.1      | Optimal Prediction in Random Fields                       | 215        |
| 5.2      | Linear Prediction—Simple and Ordinary Kriging             | 221        |
| 5.2.1    | The Mean Is Known—Simple Kriging                          | 223        |
| 5.2.2    | The Mean Is Unknown and Constant—Ordinary Kriging         | 226        |
| 5.2.3    | Effects of Nugget, Sill, and Range                        | 228        |
| 5.3      | Linear Prediction with a Spatially Varying Mean           | 232        |
| 5.3.1    | Trend Surface Models                                      | 234        |
| 5.3.2    | Localized Estimation                                      | 238        |
| 5.3.3    | Universal Kriging   | 241        |
| 5.4      | Kriging in Practice                                       | 243        |
| 5.4.1    | On the Uniqueness of the Decomposition                    | 243        |
| 5.4.2    | Local Versus Global Kriging                               | 244        |
| 5.4.3    | Filtering and Smoothing                                   | 248        |
| 5.5      | Estimating Covariance Parameters                          | 254        |
| 5.5.1    | Least Squares Estimation                                  | 256        |
| 5.5.2    | Maximum Likelihood  | 259        |
| 5.5.3    | Restricted Maximum Likelihood                             | 261        |

|          |   |            |
|----------|---|------------|
| 5.5.4    | Prediction Errors When Covariance Parameters Are Estimated  | 263        |
| 5.6      | Nonlinear Prediction  | 267        |
| 5.6.1    | Lognormal Kriging   | 267        |
| 5.6.2    | Trans-Gaussian Kriging                                      | 270        |
| 5.6.3    | Indicator Kriging   | 278        |
| 5.6.4    | Disjunctive Kriging   | 279        |
| 5.7      | Change of Support   | 284        |
| 5.7.1    | Block Kriging   | 285        |
| 5.7.2    | The Multi-Gaussian Approach                                 | 289        |
| 5.7.3    | The Use of Indicator Data                                   | 290        |
| 5.7.4    | Disjunctive Kriging and Isofactorial Models                 | 290        |
| 5.7.5    | Constrained Kriging   | 291        |
| 5.8      | On the Popularity of the Multivariate Gaussian Distribution | 292        |
| 5.9      | Chapter Problems  | 295        |
| <b>6</b> | <b>Spatial Regression Models</b>                            | <b>299</b> |
| 6.1      | Linear Models with Uncorrelated Errors                      | 301        |
| 6.1.1    | Ordinary Least Squares—Inference and Diagnostics            | 303        |
| 6.1.2    | Working with OLS Residuals                                  | 307        |
| 6.1.3    | Spatially Explicit Models                                   | 316        |
| 6.2      | Linear Models with Correlated Errors                        | 321        |
| 6.2.1    | Mixed Models  | 325        |
| 6.2.2    | Spatial Autoregressive Models                               | 335        |
| 6.2.3    | Generalized Least Squares—Inference and Diagnostics         | 341        |
| 6.3      | Generalized Linear Models                                   | 352        |
| 6.3.1    | Background  | 352        |
| 6.3.2    | Fixed Effects and the Marginal Specification                | 354        |
| 6.3.3    | A Caveat  | 355        |
| 6.3.4    | Mixed Models and the Conditional Specification              | 356        |
| 6.3.5    | Estimation in Spatial GLMs and GLMMs                        | 359        |
| 6.3.6    | Spatial Prediction in GLMs                                  | 369        |
| 6.4      | Bayesian Hierarchical Models                                | 383        |
| 6.4.1    | Prior Distributions   | 385        |
| 6.4.2    | Fitting Bayesian Models                                     | 386        |
| 6.4.3    | Selected Spatial Models                                     | 390        |
| 6.5      | Chapter Problems  | 400        |
| <b>7</b> | <b>Simulation of Random Fields</b>                          | <b>405</b> |
| 7.1      | Unconditional Simulation of Gaussian Random Fields          | 406        |
| 7.1.1    | Cholesky (LU) Decomposition                                 | 407        |
| 7.1.2    | Spectral Decomposition                                      | 407        |
| 7.2      | Conditional Simulation of Gaussian Random Fields            | 407        |
| 7.2.1    | Sequential Simulation                                       | 408        |
| 7.2.2    | Conditioning a Simulation by Kriging                        | 409        |
| 7.3      | Simulated Annealing   | 409        |

|          |  |            |
|----------|--|------------|
| 7.4      | Simulating from Convolutions   | 413        |
| 7.5      | Simulating Point Processes   | 418        |
| 7.5.1    | Homogeneous Poisson Process on the Rectangle $(0, 0) \times (a, b)$ with Intensity $\lambda$ | 418        |
| 7.5.2    | Inhomogeneous Poisson Process with Intensity $\lambda(\mathbf{s})$                           | 419        |
| 7.6      | Chapter Problems   | 419        |
| <b>8</b> | <b>Non-Stationary Covariance</b>   | <b>421</b> |
| 8.1      | Types of Non-Stationarity  | 421        |
| 8.2      | Global Modeling Approaches   | 422        |
| 8.2.1    | Parametric Models  | 422        |
| 8.2.2    | Space Deformation  | 423        |
| 8.3      | Local Stationarity   | 425        |
| 8.3.1    | Moving Windows   | 425        |
| 8.3.2    | Convolution Methods  | 426        |
| 8.3.3    | Weighted Stationary Processes  | 428        |
| <b>9</b> | <b>Spatio-Temporal Processes</b>   | <b>431</b> |
| 9.1      | A New Dimension  | 431        |
| 9.2      | Separable Covariance Functions   | 434        |
| 9.3      | Non-Separable Covariance Functions   | 435        |
| 9.3.1    | Monotone Function Approach   | 436        |
| 9.3.2    | Spectral Approach  | 436        |
| 9.3.3    | Mixture Approach   | 438        |
| 9.3.4    | Differential Equation Approach   | 439        |
| 9.4      | The Spatio-Temporal Semivariogram  | 440        |
| 9.5      | Spatio-Temporal Point Processes  | 442        |
| 9.5.1    | Types of Processes   | 442        |
| 9.5.2    | Intensity Measures   | 443        |
| 9.5.3    | Stationarity and Complete Randomness   | 444        |
|          | <b>References</b>  | <b>447</b> |
|          | <b>Author Index</b>  | <b>463</b> |
|          | <b>Subject Index</b>   | <b>467</b> |