

Contents

Preface to the First Edition	xiii
Preface to the Second Edition	xv
1 Introduction	1
1.1 Population growth	1
1.2 Administration of drugs	4
1.3 Cell division	9
1.4 Differential equations with separable variables	11
1.5 Equations of homogeneous type	14
1.6 Linear differential equations of the first order	16
1.7 Numerical solution of first-order equations	19
1.8 Symbolic computation in MATLAB [®]	24
1.9 Notes	27
2 Linear Ordinary Differential Equations with Constant Coefficients	33
2.1 Introduction	33
2.2 First-order linear differential equations	35
2.3 Linear equations of the second order	36
2.4 Finding the complementary function	37
2.5 Determining a particular integral	41
2.6 Forced oscillations	50
2.7 Differential equations of order n	52
2.8 Uniqueness	55
3 Systems of Linear Ordinary Differential Equations	61
3.1 First-order systems of equations with constant coefficients	61
3.2 Replacement of one differential equation by a system	64
3.3 The general system	66
3.4 The fundamental system	68
3.5 Matrix notation	72
3.6 Initial and boundary value problems	77
3.7 Solving the inhomogeneous differential equation	82
3.8 Numerical solution of linear boundary value problems	84

4	Modelling Biological Phenomena	91
4.1	Introduction	91
4.2	Heartbeat	91
4.3	Nerve impulse transmission	94
4.4	Chemical reactions	100
4.5	Predator–prey models	106
4.6	Notes	109
5	First-Order Systems of Ordinary Differential Equations	115
5.1	Existence and uniqueness	115
5.2	Epidemics	118
5.3	The phase plane and the Jacobian matrix	119
5.4	Local stability	121
5.5	Stability	128
5.6	Limit cycles	133
5.7	Forced oscillations	139
5.8	Numerical solution of systems of equations	143
5.9	Symbolic computation on first-order systems of equations and higher-order equations	147
5.10	Numerical solution of nonlinear boundary value problems	149
5.11	Appendix: existence theory	153
6	Mathematics of Heart Physiology	163
6.1	The local model	163
6.2	The threshold effect	166
6.3	The phase plane analysis and the heartbeat model	168
6.4	Physiological considerations of the heartbeat cycle	171
6.5	A model of the cardiac pacemaker	173
6.6	Notes	175
7	Mathematics of Nerve Impulse Transmission	177
7.1	Excitability and repetitive firing	177
7.2	Travelling waves	185
7.3	Qualitative behaviour of travelling waves	187
7.4	Piecewise linear model	190
7.5	Notes	194
8	Chemical Reactions	197
8.1	Wavefronts for the Belousov–Zhabotinskii reaction	197
8.2	Phase plane analysis of Fisher’s equation	198
8.3	Qualitative behaviour in the general case	199
8.4	Spiral waves and $\lambda - \omega$ systems	204
8.5	Notes	207

9 Predator and Prey	211
9.1 Catching fish	211
9.2 The effect of fishing	213
9.3 The Volterra–Lotka model	215
10 Partial Differential Equations	223
10.1 Characteristics for equations of the first order	223
10.2 Another view of characteristics	230
10.3 Linear partial differential equations of the second order	232
10.4 Elliptic partial differential equations	235
10.5 Parabolic partial differential equations	239
10.6 Hyperbolic partial differential equations	239
10.7 The wave equation	240
10.8 Typical problems for the hyperbolic equation	245
10.9 The Euler–Darboux equation	250
10.10 Visualisation of solutions	251
11 Evolutionary Equations	259
11.1 The heat equation	259
11.2 Separation of variables	262
11.3 Simple evolutionary equations	269
11.4 Comparison theorems	277
11.5 Notes	289
12 Problems of Diffusion	293
12.1 Diffusion through membranes	293
12.2 Energy and energy estimates	299
12.3 Global behaviour of nerve impulse transmissions	304
12.4 Global behaviour in chemical reactions	308
12.5 Turing diffusion driven instability and pattern formation	311
12.6 Finite pattern forming domains	321
12.7 Notes	325
13 Bifurcation and Chaos	329
13.1 Bifurcation	329
13.2 Bifurcation of a limit cycle	334
13.3 Discrete bifurcation and period-doubling	336
13.4 Chaos	342
13.5 Stability of limit cycles	346
13.6 The Poincaré plane	350
13.7 Averaging	355

14 Numerical Bifurcation Analysis	367
14.1 Fixed points and stability	367
14.2 Path-following and bifurcation analysis	370
14.3 Following stable limit cycles	376
14.4 Bifurcation in discrete systems	378
14.5 Strange attractors and chaos	380
14.6 Stability analysis of partial differential equations	384
14.7 Notes	385
15 Growth of Tumours	389
15.1 Introduction	389
15.2 Mathematical Model I of tumour growth	392
15.3 Spherical tumour growth based on Model I	395
15.4 Stability of tumour growth based on Model I	399
15.5 Mathematical Model II of tumour growth	401
15.6 Spherical tumour growth based on Model II	404
15.7 Stability of tumour growth based on Model II	406
15.8 Notes	407
16 Epidemics	411
16.1 The Kermack–McKendrick model	411
16.2 Vaccination	413
16.3 An incubation model	414
16.4 Spreading in space	418
Answers to Selected Exercises	427
Index	439