

BRIEF CONTENTS

Acknowledgments xv

Introduction xvii

PART I: INTRODUCTION TO PROBABILITY

Chapter 1: Bayesian Thinking and Everyday Reasoning 3

Chapter 2: Measuring Uncertainty 13

Chapter 3: The Logic of Uncertainty 21

Chapter 4: Creating a Binomial Probability Distribution 33

Chapter 5: The Beta Distribution 45

PART II: BAYESIAN PROBABILITY AND PRIOR PROBABILITIES

Chapter 6: Conditional Probability 59

Chapter 7: Bayes' Theorem with LEGO 67

Chapter 8: The Prior, Likelihood, and Posterior of Bayes' Theorem 73

Chapter 9: Bayesian Priors and Working with Probability Distributions 83

PART III: PARAMETER ESTIMATION

Chapter 10: Introduction to Averaging and Parameter Estimation 93

Chapter 11: Measuring the Spread of Our Data 103

Chapter 12: The Normal Distribution 111

Chapter 13: Tools of Parameter Estimation: The PDF, CDF, and Quantile Function 123

Chapter 14: Parameter Estimation with Prior Probabilities 137

PART IV: HYPOTHESIS TESTING: THE HEART OF STATISTICS

Chapter 15: From Parameter Estimation to Hypothesis Testing:
Building a Bayesian A/B Test 149

Chapter 16: Introduction to the Bayes Factor and Posterior Odds:
The Competition of Ideas 157

Chapter 17: Bayesian Reasoning in the Twilight Zone	167
Chapter 18: When Data Doesn't Convince You.	175
Chapter 19: From Hypothesis Testing to Parameter Estimation	183
Appendix A: A Quick Introduction to R.	195
Appendix B: Enough Calculus to Get By	215
Index	229

CONTENTS IN DETAIL

ACKNOWLEDGMENTS

XV

INTRODUCTION

XVII

Why Learn Statistics?	xviii
What Is "Bayesian" Statistics?	xix
What's in This Book	xix
Part I: Introduction to Probability	xix
Part II: Bayesian Probability and Prior Probabilities	xx
Part III: Parameter Estimation	xxi
Part IV: Hypothesis Testing: The Heart of Statistics	xxi
Background for Reading the Book	xxii
Now Off on Your Adventure!	xxii

PART I: INTRODUCTION TO PROBABILITY

1

BAYESIAN THINKING AND EVERYDAY REASONING

3

Reasoning About Strange Experiences	4
Observing Data	4
Holding Prior Beliefs and Conditioning Probabilities	5
Forming a Hypothesis	6
Spotting Hypotheses in Everyday Speech	8
Gathering More Evidence and Updating Your Beliefs	8
Comparing Hypotheses	9
Data Informs Belief; Belief Should Not Inform Data	10
Wrapping Up	11
Exercises	11

2

MEASURING UNCERTAINTY

13

What Is a Probability?	14
Calculating Probabilities by Counting Outcomes of Events	15
Calculating Probabilities as Ratios of Beliefs	16
Using Odds to Determine Probability	17
Solving for the Probabilities	17
Measuring Beliefs in a Coin Toss	18
Wrapping Up	19
Exercises	20

3 THE LOGIC OF UNCERTAINTY 21

Combining Probabilities with AND 22
 Solving a Combination of Two Probabilities 22
 Applying the Product Rule of Probability 24
 Example: Calculating the Probability of Being Late 25
Combining Probabilities with OR 26
 Calculating OR for Mutually Exclusive Events 26
 Using the Sum Rule for Non-Mutually Exclusive Events 28
 Example: Calculating the Probability of Getting a Hefty Fine 29
Wrapping Up 30
Exercises 31

4 CREATING A BINOMIAL PROBABILITY DISTRIBUTION 33

Structure of a Binomial Distribution 34
Understanding and Abstracting Out the Details of Our Problem 35
Counting Our Outcomes with the Binomial Coefficient 36
 Combinatorics: Advanced Counting with the Binomial Coefficient 37
 Calculating the Probability of the Desired Outcome 38
Example: Gacha Games 41
Wrapping Up 43
Exercises 43

5 THE BETA DISTRIBUTION 45

A Strange Scenario: Getting the Data 46
 Distinguishing Probability, Statistics, and Inference 46
 Collecting Data 46
 Calculating the Probability of Probabilities 47
The Beta Distribution 50
 Breaking Down the Probability Density Function 50
 Applying the Probability Density Function to Our Problem 51
 Quantifying Continuous Distributions with Integration 52
Reverse-Engineering the Gacha Game 53
Wrapping Up 55
Exercises 55

PART II: BAYESIAN PROBABILITY AND PRIOR PROBABILITIES

6 CONDITIONAL PROBABILITY 59

Introducing Conditional Probability 60
 Why Conditional Probabilities Are Important 60
 Dependence and the Revised Rules of Probability 61
Conditional Probabilities in Reverse and Bayes' Theorem 62

Introducing Bayes' Theorem	64
Wrapping Up	65
Exercises	66

7 BAYES' THEOREM WITH LEGO 67

Working Out Conditional Probabilities Visually	70
Working Through the Math	71
Wrapping Up	72
Exercises	72

8 THE PRIOR, LIKELIHOOD, AND POSTERIOR OF BAYES' THEOREM 73

The Three Parts	74
Investigating the Scene of a Crime	74
Solving for the Likelihood	75
Calculating the Prior	75
Normalizing the Data	76
Considering Alternative Hypotheses	78
The Likelihood for Our Alternative Hypothesis	78
The Prior for Our Alternative Hypothesis	78
The Posterior for Our Alternative Hypothesis	79
Comparing Our Unnormalized Posteriors	80
Wrapping Up	81
Exercises	81

9 BAYESIAN PRIORS AND WORKING WITH PROBABILITY DISTRIBUTIONS 83

C-3PO's Asteroid Field Doubts	84
Determining C-3PO's Beliefs	84
Accounting for Han's Badassery	85
Creating Suspense with a Posterior	87
Wrapping Up	88
Exercises	89

PART III: PARAMETER ESTIMATION

10 INTRODUCTION TO AVERAGING AND PARAMETER ESTIMATION 93

Estimating Snowfall	94
Averaging Measurements to Minimize Error	94
Solving a Simplified Version of Our Problem	95
Solving a More Extreme Case	97
Estimating the True Value with Weighted Probabilities	98
Defining Expectation, Mean, and Averaging	99

Means for Measurement vs. Means for Summary	100
Wrapping Up	101
Exercises.	101

11
MEASURING THE SPREAD OF OUR DATA **103**

Dropping Coins in a Well	104
Finding the Mean Absolute Deviation	104
Finding the Variance	106
Finding the Standard Deviation	107
Wrapping Up	109
Exercises.	109

12
THE NORMAL DISTRIBUTION **111**

Measuring Fuses for Dastardly Deeds	112
The Normal Distribution	114
Solving the Fuse Problem.	116
Some Tricks and Intuitions	118
"N Sigma" Events	120
The Beta Distribution and the Normal Distribution.	121
Wrapping Up	122
Exercises.	122

13
TOOLS OF PARAMETER ESTIMATION:
THE PDF, CDF, AND QUANTILE FUNCTION **123**

Estimating the Conversion Rate for an Email Signup List	124
The Probability Density Function.	124
Visualizing and Interpreting the PDF	125
Working with the PDF in R	126
Introducing the Cumulative Distribution Function.	127
Visualizing and Interpreting the CDF	130
Finding the Median	130
Approximating Integrals Visually.	131
Estimating Confidence Intervals.	132
Using the CDF in R	133
The Quantile Function.	133
Visualizing and Understanding the Quantile Function	134
Calculating Quantiles in R	135
Wrapping Up	135
Exercises.	136

14
PARAMETER ESTIMATION WITH PRIOR PROBABILITIES **137**

Predicting Email Conversion Rates	138
Taking in Wider Context with Priors	139
Prior as a Means of Quantifying Experience	143

Is There a Fair Prior to Use When We Know Nothing?	144
Wrapping Up	146
Exercises	146

PART IV: HYPOTHESIS TESTING: THE HEART OF STATISTICS

15 FROM PARAMETER ESTIMATION TO HYPOTHESIS TESTING: BUILDING A BAYESIAN A/B TEST 149

Setting Up a Bayesian A/B Test	150
Finding Our Prior Probability	150
Collecting Data	151
Monte Carlo Simulations	152
In How Many Worlds Is B the Better Variant?	153
How Much Better Is Each Variant B Than Each Variant A?	154
Wrapping Up	156
Exercises	156

16 INTRODUCTION TO THE BAYES FACTOR AND POSTERIOR ODDS: THE COMPETITION OF IDEAS 157

Revisiting Bayes' Theorem	158
Building a Hypothesis Test Using the Ratio of Posteriors	159
The Bayes Factor	159
Prior Odds	159
Posterior Odds	160
Wrapping Up	164
Exercises	165

17 BAYESIAN REASONING IN THE TWILIGHT ZONE 167

Bayesian Reasoning in the Twilight Zone	168
Using the Bayes Factor to Understand the Mystic Seer	168
Measuring the Bayes Factor	169
Accounting for Prior Beliefs	170
Developing Our Own Psychic Powers	171
Wrapping Up	173
Exercises	173

18 WHEN DATA DOESN'T CONVINCING YOU 175

A Psychic Friend Rolling Dice	176
Comparing Likelihoods	176
Incorporating Prior Odds	177
Considering Alternative Hypotheses	178
Arguing with Relatives and Conspiracy Theorists	179

Wrapping Up	181
Exercises.	181

19 FROM HYPOTHESIS TESTING TO PARAMETER ESTIMATION 183

Is the Carnival Game Really Fair?	184
Considering Multiple Hypotheses	186
Searching for More Hypotheses with R	186
Adding Priors to Our Likelihood Ratios.	188
Building a Probability Distribution	190
From the Bayes Factor to Parameter Estimation	191
Wrapping Up	194
Exercises.	194

A A QUICK INTRODUCTION TO R 195

R and RStudio	196
Creating an R Script.	197
Basic Concepts in R	197
Data Types.	197
Missing Values	200
Vectors	200
Functions.	201
Basic Functions	202
Random Sampling	206
The runif() Function	206
The rnorm() Function	207
The sample() Function	207
Using set.seed() for Predictable Random Results	208
Defining Your Own Functions	209
Creating Basic Plots	210
Exercise: Simulating a Stock Price	213
Summary	214

B ENOUGH CALCULUS TO GET BY 215

Functions.	216
Determining How Far You've Run	217
Measuring the Area Under the Curve: The Integral	219
Measuring the Rate of Change: The Derivative	223
The Fundamental Theorem of Calculus	227

INDEX 229