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Since the advent of the Global Navigation Satellite System (GNSS), and particularly Global Positioning System (GPS), the fields within geosciences such as geodesy, geoinformatics, geophysics, hydrology etc., have undergone tremendous changes. GPS satellites have revolutionized operations in these fields and the entire world in ways that its inventors never fathomed. The initial goal of GPS satellites was to provide the capability for the US army to position from space. This way, they could be able to know the positions of their submarines without necessarily relying on fixed ground targets which were liable to enemy attack. Slowly, but surely, the civilian community, led by geodesists, began to devise methods of exploiting the potentials of this system. The initial focus of research was on the improvement of positioning accuracies since civilians were only accessible to the coarse acquisition C/A-code of the GPS signal. This code is less precise as compared to the P-code used by the US army and its allies. The other source of error in positioning was the Selective Availability (SA). However, in May 2000, the then United States of America's president Bill Clinton officially discontinued the selective availability.

As research on GPS progressed, so were new discoveries of its uses. For example, previous research focussed on modelling or eliminating atmospheric effects such as refraction and multipath on the transmitted signals. In the last decade, however, Melbourne et al. [248] suggested that this negative effect of the atmosphere on GPS signals could be inverted to remote sense the atmosphere for vertical profiles of temperature and pressure. This gave birth to the new field of GPS meteorology which is currently an active area of research. GPS meteorology has now propelled environmental and atmospheric studies with the en-