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$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

can be constructed, which consists of only one column. Therefore it is sometimes called a *column vector*. Consider again the same small firm and assume that for the next week the management considers two alternative production plans. Let x_1, x_2, x_3 and y_1, y_2, y_3 denote the alternative production volumes. The data can be conveniently summarized in a rectangular array form:

$$\begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{pmatrix} \tag{1.1}$$

where the rows correspond to the different products, and the columns correspond to the alternative plans. This array consists of 3 rows and 2 columns, therefore it is usually called a 3×2 *matrix* (pronounced "three by