

Brief Contents

<i>Contents</i>	<i>page</i>
<i>Preface</i>	<i>xv</i>
L Linear Algebra	1
L1 Mathematics before numbers	3
L2 Vector spaces	19
L3 Euclidean geometry	38
L4 Vector product	55
L5 Linear maps	63
L6 Determinants	86
L7 Matrix diagonalization	98
L8 Unitarity and Hermiticity	109
L9 Linear algebra in function spaces	129
L10 Multilinear algebra	147
PL Problems: Linear Algebra	170
C Calculus	205
Introductory remarks	207
C1 Differentiation of one-dimensional functions	208
C2 Integration of one-dimensional functions	216
C3 Partial differentiation	229
C4 Multidimensional integration	238

C5 Taylor series	259
C6 Fourier calculus	271
C7 Differential equations	300
C8 Functional calculus	330
C9 Calculus of complex functions	338
PC Problems: Calculus	358
 V Vector Calculus	 403
Introductory remarks	405
V1 Curves	406
V2 Curvilinear coordinates	414
V3 Fields	430
V4 Introductory concepts of differential geometry	463
V5 Alternating differential forms	477
V6 Riemannian differential geometry	502
V7 Case study: differential forms and electrodynamics	518
PV Problems: Vector Calculus	533
 S Solutions	 563
SL Solutions: Linear Algebra	565
SC Solutions: Calculus	603
SV Solutions: Vector Calculus	658
 <i>Index</i>	 693

Contents

Preface	L1 Mathematics before numbers	L2 Vector spaces	L3 Euclidean geometry	L4 Vector product	L5 Linear maps	vii
	L1.1 Sets and maps	L2.1 The standard vector space \mathbb{R}^n	L3.1 Scalar product of \mathbb{R}^n	L4.1 Geometric formulation	L5.1 Linear maps	1
	L1.2 Groups	L2.2 General definition of vector spaces	L3.2 Normalization and orthogonality	L4.2 Algebraic formulation	L5.2 Matrices	3
	L1.3 Fields	L2.3 Vector spaces: examples	L3.3 Inner product spaces	L4.3 Further properties of the vector product	L5.3 Matrix multiplication	7
	L1.4 Summary and outlook	L2.4 Basis and dimension	L3.4 Complex inner product	L4.4 Summary and outlook		12
		L2.5 Vector space isomorphism	L3.5 Summary and outlook			17
		L2.6 Summary and outlook				19
L2						22
L3						25
L4						28
L5						34
						36
						38
						39
						41
						46
						52
						53
						55
						55
						58
						60
						61
						63
						63
						65
						69

L5.4	The inverse of a matrix	72
L5.5	General linear maps and matrices	78
L5.6	Matrices describing coordinate changes	80
L5.7	Summary and outlook	85
L6	Determinants	86
L6.1	Definition and geometric interpretation	86
L6.2	Computing determinants	88
L6.3	Properties of determinants	91
L6.4	Some applications	94
L6.5	Summary and outlook	96
L7	Matrix diagonalization	98
L7.1	Eigenvectors and eigenvalues	98
L7.2	Characteristic polynomial	100
L7.3	Matrix diagonalization	102
L7.4	Functions of matrices	107
L7.5	Summary and outlook	108
L8	Unitarity and Hermiticity	109
L8.1	Unitarity and orthogonality	109
L8.2	Hermiticity and symmetry	117
L8.3	Relation between Hermitian and unitary matrices	120
L8.4	Case study: linear algebra in quantum mechanics	123
L8.5	Summary and outlook	127
L9	Linear algebra in function spaces	129
L9.1	Bases of function space	130
L9.2	Linear operators and eigenfunctions	134
L9.3	Self-adjoint linear operators	137
L9.4	Function spaces with unbounded support	144
L9.5	Summary and outlook	145
L10	Multilinear algebra	147
L10.1	Direct sum and direct product of vector spaces	147
L10.2	Dual space	150
L10.3	Tensors	154
L10.4	Alternating forms	157
L10.5	Visualization of alternating forms	158
L10.6	Wedge product	161
L10.7	Inner derivative	162
L10.8	Pullback	163
L10.9	Metric structures	165
L10.10	Summary and outlook	169

PL	Problems: Linear Algebra	170
P.L1	Mathematics before numbers	170
P.L2	Vector spaces	175
P.L3	Euclidean geometry	178
P.L4	Vector product	180
P.L5	Linear maps	182
P.L6	Determinants	190
P.L7	Matrix diagonalization	191
P.L8	Unitarity and hermiticity	194
P.L10	Multilinear algebra	197
C	Calculus	205
	Introductory remarks	207
C1	Differentiation of one-dimensional functions	208
C1.1	Definition of differentiability	208
C1.2	Differentiation rules	212
C1.3	Derivatives of selected functions	213
C1.4	Summary and outlook	214
C2	Integration of one-dimensional functions	216
C2.1	The concept of integration	216
C2.2	One-dimensional integration	218
C2.3	Integration rules	222
C2.4	Practical remarks on one-dimensional integration	225
C2.5	Summary and outlook	227
C3	Partial differentiation	229
C3.1	Partial derivative	229
C3.2	Multiple partial derivatives	230
C3.3	Chain rule for functions of several variables	231
C3.4	Extrema of functions	235
C3.5	Summary and outlook	236
C4	Multidimensional integration	238
C4.1	Cartesian area and volume integrals	238
C4.2	Curvilinear area integrals	242
C4.3	Curvilinear volume integrals	249
C4.4	Curvilinear integration in arbitrary dimensions	252
C4.5	Changes of variables in higher-dimensional integration	256
C4.6	Summary and outlook	257
C5	Taylor series	259
C5.1	Taylor expansion	259

C5.2	Complex Taylor series	263
C5.3	Finite-order expansions	266
C5.4	Solving equations by Taylor expansion	267
C5.5	Higher-dimensional Taylor series	269
C5.6	Summary and outlook	270
C6	Fourier calculus	271
C6.1	The δ -function	272
C6.2	Fourier series	277
C6.3	Fourier transform	285
C6.4	Case study: frequency comb for high-precision measurements	296
C6.5	Summary and outlook	299
C7	Differential equations	300
C7.1	Typology of differential equations	301
C7.2	Separable differential equations	302
C7.3	Linear first-order differential equations	306
C7.4	Systems of linear first-order differential equations	308
C7.5	Linear higher-order differential equations	313
C7.6	General higher-order differential equations	321
C7.7	Linearizing differential equations	326
C7.8	Partial differential equations	327
C7.9	Summary and outlook	328
C8	Functional calculus	330
C8.1	Definitions	330
C8.2	Functional derivative	332
C8.3	Euler–Lagrange equations	333
C8.4	Summary and outlook	337
C9	Calculus of complex functions	338
C9.1	Holomorphic functions	338
C9.2	Complex integration	341
C9.3	Singularities	345
C9.4	Residue theorem	348
C9.5	Essential singularities	353
C9.6	Riemann surfaces	355
C9.7	Summary and outlook	357
PC	Problems: Calculus	358
P.C1	Differentiation of one-dimensional functions	358
P.C2	Integration of one-dimensional functions	361
P.C3	Partial differentiation	365
P.C4	Multidimensional integration	367
P.C5	Taylor series	376

P.C6	Fourier calculus	379
P.C7	Differential equations	385
P.C8	Functional calculus	395
P.C9	Calculus of complex functions	398
V Vector Calculus		403
Introductory remarks		405
V1	Curves	406
V1.1	Definition	406
V1.2	Curve velocity	407
V1.3	Curve length	409
V1.4	Line integral	412
V1.5	Summary and outlook	413
V2	Curvilinear coordinates	414
V2.1	Polar coordinates	414
V2.2	Coordinate basis and local basis	416
V2.3	Cylindrical and spherical coordinates	421
V2.4	A general perspective of coordinates	425
V2.5	Local coordinate bases and linear algebra	426
V2.6	Summary and outlook	429
V3	Fields	430
V3.1	Definition of fields	430
V3.2	Scalar fields	432
V3.3	Extrema of functions with constraints	439
V3.4	Gradient fields	441
V3.5	Sources of vector fields	446
V3.6	Circulation of vector fields	454
V3.7	Practical aspects of three-dimensional vector calculus	459
V3.8	Summary and outlook	461
V4	Introductory concepts of differential geometry	463
V4.1	Differentiable manifolds	464
V4.2	Tangent space	468
V4.3	Summary and outlook	476
V5	Alternating differential forms	477
V5.1	Cotangent space and differential one-forms	477
V5.2	Pushforward and pullback	481
V5.3	Forms of higher degree	487
V5.4	Integration of forms	495
V5.5	Summary and outlook	501

V6	Riemannian differential geometry	502
V6.1	Definition of the metric on a manifold	502
V6.2	Volume form and Hodge star	505
V6.3	Vectors vs. one-forms vs. two-forms in \mathbb{R}^3	507
V6.4	Case study: metric structures in general relativity	512
V6.5	Summary and outlook	517
V7	Case study: differential forms and electrodynamics	518
V7.1	The ingredients of electrodynamics	519
V7.2	Laws of electrodynamics I: Lorentz force	522
V7.3	Laws of electrodynamics II: Maxwell equations	525
V7.4	Invariant formulation	529
V7.5	Summary and outlook	532
PV	Problems: Vector Calculus	533
P.V1	Curves	533
P.V2	Curvilinear coordinates	535
P.V3	Fields	537
P.V4	Introductory concepts of differential geometry	550
P.V5	Alternating differential forms	553
P.V6	Riemannian differential geometry	559
P.V7	Differential forms and electrodynamics	560
S	Solutions	563
SL	Solutions: Linear Algebra	565
S.L1	Mathematics before numbers	565
S.L2	Vector spaces	569
S.L3	Euclidean geometry	573
S.L4	Vector product	575
S.L5	Linear Maps	578
S.L6	Determinants	585
S.L7	Matrix diagonalization	586
S.L8	Unitarity and Hermiticity	592
S.L10	Multilinear algebra	596
SC	Solutions: Calculus	603
S.C1	Differentiation of one-dimensional functions	603
S.C2	Integration of one-dimensional functions	605
S.C3	Partial differentiation	613
S.C4	Multidimensional integration	614
S.C5	Taylor series	623
S.C6	Fourier calculus	628
S.C7	Differential equations	637

S.C8	Functional calculus	649
S.C9	Calculus of complex functions	651
SV	Solutions: Vector Calculus	658
S.V1	Curves	658
S.V2	Curvilinear coordinates	660
S.V3	Fields	663
S.V4	Introductory concepts of differential geometry	680
S.V5	Alternating differential forms	683
S.V6	Riemannian differential geometry	689
S.V7	Differential forms and electrodynamics	690

Index	693
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The book is an introduction to mathematics for beginner physics students. It contains the material required in the undergraduate curriculum. The main feature distinguishing it from the large number of available books on the subject is that mathematical concepts and methods are presented in unison and on an equal footing. Let us explain what is meant by this statement.

Physics teaching mathematics often focus on the *training of methods*. They provide recipes for the algebraic manipulation of vectors and matrices, the differentiation of functions, the computation of integrals etc. Such pragmatic approaches are often justified by citing physics: physics requires advanced mathematical methodology and students have to learn this quickly as possible.

Unfortunately, knowledge of computational methods alone will not carry a student through the physics curriculum. Equally important, she needs to understand the mathematical principles and concepts behind the machinery. For example, the methodological knowledge that the derivative of x^2 equals $2x$ remains hollow, unless the conceptual meaning of this *derivative* as local linear approximation of a parabola is fully appreciated. Similar things can be said about any of the advanced mathematical methods required in academic physics teaching.

Recognizing this point, physics curricula often include lecture courses in pure mathematics. Why would be better authorized to teach mathematical concepts than mathematicians themselves? However, there is a catch: mathematicians approach the conceptual framework of their discipline from a perspective different from that of physicists. Rigorous proofs and existence theorems stand in the foreground and are more important than the communication of concepts relevant to the understanding of applications in physics. This dependency of perspective may vary when mathematicians teach "mathematics for physicists".

For these reasons, the traditional division – physics courses focusing on methods, mathematics courses on proofs – is not ideal.

Pedagogical strategy – unified presentation of concepts and methods

This book aims to bridge the divide between *unified presentation of concepts and methods*, written from the perspective of theoretical physicists. Mathematical structures are employed and introduced as an *organizing framework* for the understanding of methods. Although less emphasis is put on formal proofs, the text maintains a fair level of