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$\langle x \rangle$  Average value of  $x$ .

$\Omega$  Number of combinations (or states) of  $x$  objects taken from a total of  $N$  objects.

$N(\mu, \sigma^2)$  Normal distribution of  $x$ , with mean  $\mu$  and variance  $\sigma^2$ .

$N(\mu, P)$  Multi-dimensional normal distribution of  $x$ , with mean  $\mu$  and covariance matrix  $P$ .

$\hat{e}_i$  Basis vector for coordinates  $q_i$ .

$\hat{e}_i$  Unit-length basis vector for coordinates  $q_i$ .

$dx$  Infinitesimal, an "exact differential" of state variable  $x$ .

$dA$  Infinitesimal, an "inexact differential" of quantity  $A$  that is not a state variable.

$\rho(x)$  Linear mass density as a function of position  $x$ .

$M$  Molar mass.

$L$  Representation of  $L$  in system  $S$ .

**Chapter 2: Accessible States and the Fundamental Postulates of Statistical Mechanics**

$\Omega$  Number of internal variables in which a particle can store its energy.

$\Omega(E)$  Number of microstates that each have energy  $E$ .

$\Omega_{\text{shell}}(E)$  Total number of microstates that each have energy somewhere in the range  $E$  to  $E + \Delta E$ .

$\nu$  Number of quadratic energy terms of a particle, meaning the number of quadratic coordinates that describe the particle's energy. (In other texts, this is called the number of degrees of