

# Theoretical Foundations of Functional Data Analysis, with an Introduction to Linear Operators

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## **Provides essential coverage of functional data analysis and related areas**

This book provides a uniquely broad compendium of the key mathematical concepts and results that are relevant to the theoretical development of functional data analysis (FDA).

The self-contained treatment of selected topics of functional analysis and operator theory includes reproducing kernel Hilbert spaces, singular value decomposition of compact operators on Hilbert spaces, and perturbation theory for both self-adjoint and non-self-adjoint operators. The probabilistic foundation for FDA is described from the perspective of random elements in Hilbert spaces as well as from continuous time stochastic processes. Nonparametric estimation approaches including kernel and regularized smoothing are also introduced. These tools are then used to investigate the properties of estimators for mean elements, covariance operators, principle components, regression functions, and canonical correlations. A general treatment of canonical correlations in Hilbert spaces naturally leads to FDA formulations of factor analysis, regression, MANOVA, and discriminant analysis.

### *Key features:*

- Provides a concise but rigorous account of the theoretical background of FDA
- Introduces topics in various areas of mathematics, probability, and statistics from the perspective of FDA
- Presents a systematic exposition of the fundamental statistical issues in FDA
- Develops all material from first principles, assuming no prior knowledge of linear operators or FDA

This book is a valuable reference for statisticians and other researchers interested in developing or understanding the mathematical aspects of FDA. It is also suitable for a graduate level special topics course.

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