

# Contents

<b>List of Examples</b>	<b>xv</b>
<b>List of Principal Results</b>	<b>xix</b>
<b>List of Figures</b>	<b>xxiii</b>
<b>1. A Single Explanatory Variable</b>	<b>1</b>
1.1. Introduction, 1	
1.1.1. Ordinary Least Squares and Measurement Error, 1	
1.1.2. Estimation with Known Reliability Ratio, 5	
1.1.3. Identification, 9	
1.2. Measurement Variance Known, 13	
1.2.1. Introduction and Estimators, 13	
1.2.2. Sampling Properties of the Estimators, 15	
1.2.3. Estimation of True $x$ Values, 20	
1.2.4. Model Checks, 25	
1.3. Ratio of Measurement Variances Known, 30	
1.3.1. Introduction, 30	
1.3.2. Method of Moments Estimators, 30	
1.3.3. Least Squares Estimation, 36	
1.3.4. Tests of Hypotheses for the Slope, 44	
1.4. Instrumental Variable Estimation, 50	
1.5. Factor Analysis, 59	
1.6. Other Methods and Models, 72	
1.6.1. Distributional Knowledge, 72	

1.6.2.	The Method of Grouping, 73	
1.6.3.	Measurement Error and Prediction, 74	
1.6.4.	Fixed Observed $X$ , 79	
Appendix 1.A.	Large Sample Approximations, 85	
Appendix 1.B.	Moments of the Normal Distribution, 88	
Appendix 1.C.	Central Limit Theorems for Sample Moments, 89	
Appendix 1.D.	Notes on Notation, 95	
<b>2.</b>	<b>Vector Explanatory Variables</b>	<b>100</b>
2.1.	Bounds for Coefficients, 100	
2.2.	The Model with an Error in the Equation, 103	
2.2.1.	Estimation of Slope Parameters, 103	
2.2.2.	Estimation of True Values, 113	
2.2.3.	Higher-Order Approximations for Residuals and True Values, 118	
2.3.	The Model with No Error in the Equation, 124	
2.3.1.	The Functional Model, 124	
2.3.2.	The Structural Model, 139	
2.3.3.	Higher-Order Approximations for Residuals and True Values, 140	
2.4.	Instrumental Variable Estimation, 148	
2.5.	Modifications to Improve Moment Properties, 163	
2.5.1.	An Error in the Equation, 164	
2.5.2.	No Error in the Equation, 173	
2.5.3.	Calibration, 177	
Appendix 2.A.	Language Evaluation Data, 181	
<b>3.</b>	<b>Extensions of the Single Relation Model</b>	<b>185</b>
3.1.	Nonnormal Errors and Unequal Error Variances, 185	
3.1.1.	Introduction and Estimators, 186	
3.1.2.	Models with an Error in the Equation, 193	
3.1.3.	Reliability Ratios Known, 199	
3.1.4.	Error Variance Functionally Related to Observations, 202	
3.1.5.	The Quadratic Model, 212	
3.1.6.	Maximum Likelihood Estimation for Known Error Covariance Matrices, 217	

3.2. Nonlinear Models with No Error in the Equation, 225	
3.2.1. Introduction, 225	
3.2.2. Models Linear in $\mathbf{x}$ , 226	
3.2.3. Models Nonlinear in $\mathbf{x}$ , 229	
3.2.4. Modifications of the Maximum Likelihood Estimator, 247	
3.3. The Nonlinear Model with an Error in the Equation, 261	
3.3.1. The Structural Model, 261	
3.3.2. General Explanatory Variables, 263	
3.4. Measurement Error Correlated with True Value, 271	
3.4.1. Introduction and Estimators, 271	
3.4.2. Measurement Error Models for Multinomial Random Variables, 272	
Appendix 3.A. Data for Examples, 281	
<b>4. Multivariate Models</b>	<b>292</b>
4.1. The Classical Multivariate Model, 292	
4.1.1. Maximum Likelihood Estimation, 292	
4.1.2. Properties of Estimators, 303	
4.2. Least Squares Estimation of the Parameters of a Covariance Matrix, 321	
4.2.1. Least Squares Estimation, 321	
4.2.2. Relationships between Least Squares and Maximum Likelihood, 333	
4.2.3. Least Squares Estimation for the Multivariate Functional Model, 338	
4.3. Factor Analysis, 350	
4.3.1. Introduction and Model, 350	
4.3.2. Maximum Likelihood Estimation, 353	
4.3.3. Limiting Distribution of Factor Estimators, 360	
Appendix 4.A. Matrix-Vector Operations, 382	
Appendix 4.B. Properties of Least Squares and Maximum Likelihood Estimators, 396	
Appendix 4.C. Maximum Likelihood Estimation for Singular Measurement Covariance, 404	
<b>Bibliography</b>	<b>409</b>
<b>Author Index</b>	<b>433</b>
<b>Subject Index</b>	<b>435</b>