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The basic terminology and notation are adopted from Porter and Stirling (1993), Baker (1978), and Mikhailin and Smolitskiy (1967). Generally, while discussing eigenvalue problems the terms 'eigenvalues' and 'characteristic values' are used interchangeably. But in the literature on integral equations we find that these terms