

CONTENTS

	Preface	vii
	Notes to the Reader	xi
PART I	Riemannian Geometry	1
CHAPTER 1	Affine Differential Geometry	1
1.1	Differentiable manifolds	2
1.2	Vector fields	3
1.3	Differential forms	5
1.4	Maps	7
1.5	Lie groups	9
1.6	The frame bundle: parallelism and geodesics.	14
1.7	Curvature, torsion and the structure equations	23
1.8	Covering spaces	31
1.9	The Cartan-Ambrose-Hicks Theorem	42
CHAPTER 2	Riemannian Curvature	45
2.1	The Levi-Civita connection	46
2.2	Sectional curvature	52
2.3	Isometries and curvature	57
2.4	Models for spaces of constant curvature	62
2.5	The 2-dimensional space forms	74
2.6	Finite rotation groups	83
2.7	Homogeneous space forms	88
2.8	Appendix: The metric space structure of a riemannian manifold	91
PART II	The Euclidean Space Form Problem	97
CHAPTER 3	Flat Riemannian Manifolds	98
3.1	Discontinuous groups on euclidean space	98
3.2	The Bieberbach Theorems on crystallographic groups	100
3.3	Application to euclidean space forms	105
3.4	Questions of holonomy	107
3.5	Three dimensional euclidean space forms	111
3.6	Three attacks on the classification problem for flat compact manifolds	124
3.7	Flat homogeneous pseudo-riemannian manifolds	131

PART III	The Spherical Space Form Problem	137
CHAPTER 4	Representations of Finite Groups	138
4.1	Basic definitions	138
4.2	The Frobenius-Schur relations	139
4.3	Frobenius reciprocity and the group algebra	141
4.4	Divisibility	145
4.5	Tensor products and dual representations	147
4.6	Two lemmas on representations over algebraically non-closed fields	150
4.7	Unitary and orthogonal representations	151
CHAPTER 5	Vincent's Work on the Spherical Space Form Problem	154
5.1	Vincent's program	154
5.2	Preliminaries on p -groups	156
5.3	Necessary conditions on fixed point free groups	159
5.4	Classification of the simplest type of fixed point free groups	162
5.5	Representations of finite groups in which every Sylow subgroup is cyclic	165
5.6	A partial solution to the spherical space form problem	171
CHAPTER 6	The Classification of Fixed Point Free Groups	172
6.1	Zassenhaus' work on solvable groups with cyclic odd Sylow subgroups	173
6.2	The binary icosahedral group	181
6.3	Non-solvable fixed point free groups	195
CHAPTER 7	The Solution to the Spherical Space Form Problem	198
7.1	Representations of binary polyhedral groups	198
7.2	Fixed point free complex representations	203
7.3	The action of automorphisms on representations	211
7.4	The classification of spherical space forms	218
7.5	Spherical space forms of low dimension	224
7.6	Clifford translations	227
PART IV	Space Form Problems on Symmetric Spaces	231
CHAPTER 8	Riemannian Symmetric Spaces	231
8.1	Lie formulation of locally symmetric spaces	232
8.2	Structure of orthogonal involutive Lie algebras	234
8.3	Globally symmetric spaces and orthogonal involutive Lie algebras	240
8.4	Curvature	245

8.5	Cohomology	247
8.6	Cartan subalgebras, rank and maximal tori	252
8.7	Hermitian symmetric spaces	257
8.8	The full group of isometries	263
8.9	Extended Schläfli-Dynkin diagrams	264
8.10	Subgroups of maximal rank	275
8.11	The classification of symmetric spaces	286
8.12	Two point homogeneous spaces	293
8.13	Appendix: Manifolds with irreducible linear isotropy group	300
CHAPTER 9	Space Forms of Irreducible Symmetric Spaces	303
9.1	Feasibility of space form problems	304
9.2	Grassmann manifolds as symmetric spaces	306
9.3	Grassmann manifolds of even dimension	307
9.4	Grassmann manifolds of odd dimension	314
9.5	Symmetric spaces of positive characteristic	319
9.6	An isolated manifold	325
CHAPTER 10	Locally Symmetric Spaces of Non-negative Curvature	328
10.1	The structure theorems	329
10.2	Application of the structure theorems	333
PART V	Space Form Problems on Indefinite Metric Manifolds	337
CHAPTER 11	Spaces of Constant Curvature	337
11.1	The classification of finite space forms	338
11.2	The geometry of pseudo-spherical space forms	341
11.3	Homogeneous finite space forms	347
11.4	The lattice space forms	354
11.5	A wild Lorentz signature	366
11.6	The classification for homogeneous manifolds of constant curvature	370
CHAPTER 12	Locally Isotropic Manifolds	374
12.1	Reductive Lie groups	374
12.2	Examples of locally isotropic manifolds	380
12.3	Structure of locally isotropic spaces	385
12.4	A partial classification of complete locally isotropic manifolds	389
	References	396
	Index	403