Contents

Preface vii
Reading Guide xi

1. Stochastic Convergence 1

1.1. Introduction 2
1.2. Outer Integrals and Measurable Majorants 6
1.3. Weak Convergence 16
1.4. Product Spaces 29
1.5. Spaces of Bounded Functions 34
1.6. Spaces of Locally Bounded Functions 43
1.7. The Ball Sigma-Field and Measurability of Suprema 45
1.8. Hilbert Spaces 49
1.9. Convergence: Almost Surely and in Probability 52
1.10. Convergence: Weak, Almost Uniform, and in Probability . . . 57
1.11. Refinements 67
1.12. Uniformity and Metrization 71
Notes 75

2. Empirical Processes 79

2.1. Introduction 80
2.1.1. Overview of Chapters 2.3–2.14 83
2.1.2. Asymptotic Equicontinuity 89
2.1.3. Maximal Inequalities 90
*2.1.4. The Central Limit Theorem in Banach Spaces 91
2.2. Maximal Inequalities and Covering Numbers 95
2.2.1. Sub-Gaussian Inequalities 100
2.2.2. Bernstein's Inequality 102
*2.2.3. Tightness Under an Increment Bound 104
2.3. Symmetrization and Measurability 107
2.3.1. Symmetrization 107
*2.3.2. More Symmetrization 111
*2.3.3. Separable Versions 115
2.4. Glivenko-Cantelli Theorems 122
2.5. Donsker Theorems 127
2.5.1. Uniform Entropy 127
2.5.2. Bracketing 129
2.6. Uniform Entropy Numbers 134
2.6.1. VC-Classes of Sets 134
2.6.2. VC-Classes of Functions 140
2.6.3. Convex Hulls and VC-Hull Classes 142
2.6.4. VC-Major Classes 145
2.6.5. Examples and Permanence Properties 146
2.7. Bracketing Numbers 154
2.7.1. Smooth Functions and Sets 154
2.7.2. Monotone Functions 159
2.7.3. Closed Convex Sets and Convex Functions 162
2.7.4. Classes That Are Lipschitz in a Parameter 164
2.8. Uniformity in the Underlying Distribution 166
2.8.1. Glivenko-Cantelli Theorems 166
2.8.2. Donsker Theorems 168
2.8.3. Central Limit Theorem Under Sequences 173
2.9. Multiplier Central Limit Theorems 176
2.10. Permanence of the Donsker Property 190
2.10.1. Closures and Convex Hulls 190
2.10.2. Lipschitz Transformations 192
2.10.3. Permanence of the Uniform Entropy Bound 198
2.10.4. Partitions of the Sample Space 200
2.11. The Central Limit Theorem for Processes 205
2.11.1. Random Entropy 205
2.11.2. Bracketing 210
2.11.3. Classes of Functions Changing with n 220
2.12. Partial-Sum Processes 225

2.12.1. The Sequential Empirical Process 225
2.12.2. Partial-Sum Processes on Lattices 228
2.13. Other Donsker Classes 232
2.13.1. Sequences 232
2.13.2. Elliptical Classes 233
2.13.3. Classes of Sets 236
2.14. Tail Bounds 238
2.14.1. Finite Entropy Integrals 238
2.14.2. Uniformly Bounded Classes 245
2.14.3. Deviations from the Mean 254
2.14.4. Proof of Theorem 2.14.13 257
Notes 269

3. Statistical Applications 277

3.1. Introduction 278
3.2. M-Estimators 284
3.2.1. The Argmax Theorem 285
3.2.2. Rate of Convergence 289
3.2.3. Examples 294
3.2.4. Linearization 300
3.3. Z-Estimators 309
3.4. Rates of Convergence 321
3.4.1. Maximum Likelihood 326
3.4.2. Concave Parametrizations 330
3.4.3. Least Squares Regression 331
3.4.4. Least-Absolute-Deviation Regression 336
3.5. Random Sample Size, Poissonization and Kac Processes . . . 339
3.5.1. Random Sample Size 339
3.5.2. Poissonization 341
3.6. The Bootstrap 345
3.6.1. The Empirical Bootstrap 345
3.6.2. The Exchangeable Bootstrap 353
3.7. The Two-Sample Problem 360
3.7.1. Permutation Empirical Processes 362
3.7.2. Two-Sample Bootstrap 365
3.8. Independence Empirical Processes 367
3.9. The Delta-Method 372
3.9.1. Main Result 372
3.9.2. Gaussian Limits 376
3.9.3. The Delta-Method for the Bootstrap 377
3.9.4. Examples of the Delta-Method 381
3.10. Contiguity 401
3.10.1. The Empirical Process 406

3.10.2. Change-Point Alternatives 408
3.11. Convolution and Minimax Theorems 412
3.11.1. Efficiency of the Empirical Distribution 420
Notes 423

A. Appendix 429

A.1. Inequalities 430
A.2. Gaussian Processes 437
A.2.1. Inequalities and Gaussian Comparison 437
A.2.2. Exponential Bounds 442
A.2.3. Majorizing Measures 445
A.2.4. Further Results 447
A.3. Rademacher Processes 449
A.4. Isoperimetric Inequalities for Product Measures 451
A.5. Some Limit Theorems 456
A.6. More Inequalities 459
A.6.1. Binomial Random Variables 459
A.6.2. Multinomial Random Vectors 462
A.6.3. Rademacher Sums 463
Notes 465

References 467
Author Index 487
Subject Index 493
List of Symbols 506