Contents

CHAPTER 0. SETS AND NUMBERS

0.1 Rudiments of Logic 1
0.2 Fundamentals of Set Description 5
0.3 Set Inclusion and Equality 5
0.4 An Axiom System for Set Theory 6
0.5 Unions and Intersections 6
0.6 Set Difference 7
0.7 Integers and Induction 7
0.8 Simple Cartesian Products 11
0.9 Relations 11
0.10 Functions 12
0.11 Sequences 14
0.12 Indexing Sets 15
0.13 Important Formulas 16
0.14 Inverse Functions 16
0.15 More Important Formulas 17
0.16 Partitions 18
0.17 Equivalence Relations, Partitions and Functions 18
0.18 General Cartesian Products 19
0.19 The Sixth Axiom (Axiom of Choice) 20
0.20 Well- Orders and Zorn 21
0.21 Yet More Important Formulas 22
0.22 Cardinality 22

CHAPTER 1. METRIC AND TOPOLOGICAL SPACES

1.1 Metrics and Topologies 31
1.2 Time out for Notation 33
1.3 Metrics Generate Topologies 35
1.4 Continuous Functions 36
1.5 Subspaces 39
1.6 Comparable Topologies 39

CHAPTER 2. FROM OLD TO NEW SPACES

2.1 Product Spaces 47
2.2 Product Metrics and Topologies 51

2.3 Quotient Spaces 53
2.4 Applications (Möbius Band, Klein Bottle, Torus, Projective Plane, etc.) 55

CHAPTER 3. VERY SPECIAL SPACES

3.1 Compact Spaces 67
3.2 Compactification (One-Point Only) 73
3.3 Complete Metric Spaces (Baire-Category, Banach Contraction Theorem and Applications of Roots of $y = h(x)$ to Systems of Linear Equations 75
3.4 Connected and Arcwise Connected Spaces 80

CHAPTER 4. FUNCTION SPACES

4.1 Function Space Topologies 89
4.2 Completness and Compactness (Ascoli-Arzela Theorem, Picard's Theorem, Peano's Theorem) 92
4.3 Approximation (Bernstein's polynomials, Stone-Weierstrass Approximation) 100
4.4 Function-Space Functions 103

CHAPTER 5. TOPOLOGICAL GROUPS

5.1 Elementary Structures 114
5.2 Topological Isomorphism Theorems 121
5.3 Quotient Group Recognition 123
5.4 Morphism Groups (Topological and Transformation Groups) 124

CHAPTER 6. SPECIAL GROUPS

6.1 Preliminaries 131
6.2 Groups of Matrices 134
6.3 Groups of Isometries 135
6.4 Relativity and Lorentz Transformations 140

CHAPTER 7. NORMALITY AND PARACOMPACTNESS

7.1 Normal Spaces (Urysohn's Lemma) 147
7.2 Paracompact Spaces (Partitions of Unity with and Application to Embedding of Manifolds in Euclidean Spaces) 151

CHAPTER 8. THE FUNDAMENTAL GROUP

8.1 Description of $\Pi_1(X,b)$ 167
8.2 Elementary Facts about $\Pi_1(X,b)$ 173
8.3 Simplicial Complexes 175
8.4 Barycentric Subdivision 179
8.5 The Simplicial Approximation 181
8.6 The Fundamental Group of Polytopes 183
8.7 Graphs and Trees 187

APPENDIX A. SOME INEQUALITIES 193
APPENDIX B. BINOMIAL EQUALITIES 195
LIST OF SYMBOLS 197

INDEX 199