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We assume that the reader has a good grasp of the fundamentals of Logic and Set Theory, even though a rather succinct review appears in the introductory chapter 0. The reader should also be very familiar with elementary analysis. Some familiarity with Group Theory is required for chapters 5, 6 and 8.

The problems which appear at the end of each chapter not only provide ample opportunity for applying the concepts and techniques just learned but also are used to introduce additional concepts and techniques which complement the text and point to further study elsewhere.

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