Contents

PREFACE v

CHAPTER 1 The Algebra and Topology of $\mathbb{R}^n$ 1

§1. Review of Linear Algebra 1
§2. Matrix Inversion and Determinants 11
§3. Review of Topology in $\mathbb{R}^n$ 25
§4. Compact Subspaces and Connected Subspaces of $\mathbb{R}^n$ 32

CHAPTER 2 Differentiation 41

§5. The Derivative 41
§6. Continuously Differentiable Functions 49
§7. The Chain Rule 56
§8. The Inverse Function Theorem 63
*§9. The Implicit Function Theorem 71

CHAPTER 3 Integration 81

§10. The Integral over a Rectangle 81
§11. Existence of the Integral 91
§12. Evaluation of the Integral 98
§13. The Integral over a Bounded Set 104
§14. Rectifiable Sets 112
§15. Improper Integrals 121

CHAPTER 4 Change of Variables 135

§16. Partitions of Unity 136
§17. The Change of Variables Theorem 144
§18. Diffeomorphisms in $\mathbb{R}^n$ 152
§19. Proof of the Change of Variables Theorem 161
§20. Applications of Change of Variables 169

CHAPTER 5 Manifolds 179

§21. The Volume of a Parallelopiped 180
§22. The Volume of a Parametrized-Manifold 188
§23. Manifolds in $\mathbb{R}^n$ 196
§24. The Boundary of a Manifold 203
§25. Integrating a Scalar Function over a Manifold 209

CHAPTER 6 Differential Forms 219

§26. Multilinear Algebra 220
§27. Alternating Tensors 226
§28. The Wedge Product 236
§29. Tangent Vectors and Differential Forms 244
§30. The Differential Operator 252
*§31. Application to Vector and Scalar Fields 262
§32. The Action of a Differentiable Map 267

CHAPTER 7 Stokes' Theorem 275

§33. Integrating Forms over Parametrized-Manifolds 275
§34. Orientable Manifolds 281
§35. Integrating Forms over Oriented Manifolds 293
*§36. A Geometric Interpretation of Forms and Integrals 297
§37. The Generalized Stokes' Theorem 301
*§38. Applications to Vector Analysis 310

CHAPTER 8 Closed Forms and Exact Forms 323

§39. The Poincaré Lemma 324
§40. The deRham Groups of Punctured Euclidean Space 334

CHAPTER 9 Epilogue—Life Outside $\mathbf{R}^n$ 345

§41. Differentiable Manifolds and Riemannian Manifolds 345

BIBLIOGRAPHY 359

INDEX 361