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This book provides an introduction to the growing field of ergodic theory, also known as measurable dynamics. It covers topics such as recurrence, ergodicity, the ergodic theorem and mixing. It is aimed at students who have completed a basic course in undergraduate real analysis covering topics such as basic compactness properties and open and closed sets in the real line. Measure theory is not assumed and is developed as needed. Readers less familiar with these topics will find a discussion of the relevant material from real analysis in the appendices.

I have used early versions of this book in courses that are designed as capstone courses for the mathematics major, including students with a variety of interests and backgrounds. The study of measurable dynamics can be used to reinforce and apply the student's knowledge of measure theory and real analysis while introducing some beautiful mathematics of relatively recent vintage. Measure theory is developed as needed and applied to study notions in dynamics. While it has less emphasis, some metric space topology, including the Baire category theorem, is presented and applied to topological dynamics. Several examples are developed in detail to illustrate concepts from measurable and topological dynamics.

This book can be used as a special-topics course for upper-level mathematics students. It can also be used as a short introduction