

Contents

Preface	vii
1. The Basics	1
1.1 Graphs*	2
1.2 The degree of a vertex*	5
1.3 Paths and cycles*	6
1.4 Connectivity*	10
1.5 Trees and forests*	13
1.6 Bipartite graphs*	17
1.7 Contraction and minors*	19
1.8 Euler tours*	22
1.9 Some linear algebra	23
1.10 Other notions of graphs	27
Exercises	30
Notes	33
2. Matching, Covering and Packing	35
2.1 Matching in bipartite graphs*	36
2.2 Matching in general graphs ^(*)	41
2.3 The Erdős-Pósa theorem	45
2.4 Tree packing and arboricity	48
2.5 Path covers	52
Exercises	53
Notes	56

* Sections marked by an asterisk are recommended for a first course.
Of sections marked ^(*), the beginning is recommended for a first course.

3. Connectivity	59
3.1 2-Connected graphs and subgraphs*	59
3.2 The structure of 3-connected graphs ^(*)	62
3.3 Menger's theorem*	67
3.4 Mader's theorem	72
3.5 Linking pairs of vertices ^(*)	74
Exercises	82
Notes	85
4. Planar Graphs	89
4.1 Topological prerequisites*	90
4.2 Plane graphs*	92
4.3 Drawings	98
4.4 Planar graphs: Kuratowski's theorem*	102
4.5 Algebraic planarity criteria	107
4.6 Plane duality	110
Exercises	113
Notes	117
5. Colouring	119
5.1 Colouring maps and planar graphs*	120
5.2 Colouring vertices*	122
5.3 Colouring edges*	127
5.4 List colouring	129
5.5 Perfect graphs	135
Exercises	142
Notes	146
6. Flows	149
6.1 Circulations ^(*)	150
6.2 Flows in networks*	151
6.3 Group-valued flows	154
6.4 k -Flows for small k	159
6.5 Flow-colouring duality	162
6.6 Tutte's flow conjectures	165
Exercises	169
Notes	171

7. Extremal Graph Theory	173
7.1 Subgraphs*	174
7.2 Minors ^(*)	180
7.3 Hadwiger's conjecture*	183
7.4 Szemerédi's regularity lemma	187
7.5 Applying the regularity lemma	195
Exercises	201
Notes	204
8. Infinite Graphs	209
8.1 Basic notions, facts and techniques*	210
8.2 Paths, trees, and ends ^(*)	219
8.3 Homogeneous and universal graphs*	228
8.4 Connectivity and matching	231
8.5 Recursive structures	242
8.6 Graphs with ends: the complete picture	245
8.7 The topological cycle space	254
8.8 Infinite graphs as limits of finite ones	258
Exercises	261
Notes	273
9. Ramsey Theory for Graphs	283
9.1 Ramsey's original theorems*	284
9.2 Ramsey numbers ^(*)	287
9.3 Induced Ramsey theorems	290
9.4 Ramsey properties and connectivity ^(*)	300
Exercises	303
Notes	304
10. Hamilton Cycles	307
10.1 Sufficient conditions*	307
10.2 Hamilton cycles and degree sequences	311
10.3 Hamilton cycles in the square of a graph	314
Exercises	319
Notes	320

11. Random Graphs	323
11.1 The notion of a random graph*	324
11.2 The probabilistic method*	329
11.3 Properties of almost all graphs*	332
11.4 Threshold functions and second moments	335
Exercises	342
Notes	344
12. Graph Minors	347
12.1 Well-quasi-ordering ^(*)	348
12.2 The graph minor theorem for trees	349
12.3 Tree-decompositions ^(*)	351
12.4 Tree-width ^(*)	355
12.5 Tangles	360
12.6 Tree-decompositions and forbidden minors	369
12.7 The graph minor theorem ^(*)	374
Exercises	382
Notes	388
A. Infinite sets	393
B. Surfaces	399
Hints for all the exercises.....	407
Index.....	409
Symbol index	427