Contents

1 Introduction 1
1.1 A Motivating Example 1
1.2 Probability and Quantile Transformations 3
1.3 Copulas 5
1.4 Structure and Philosophy of the Book 6
1.5 Additional Reading 7
References 8

2 Copulas 9
2.1 Definition and Characterization 9
2.2 The Fréchet–Hoeffding Bounds 18
2.3 Sklar's Theorem 22
2.4 The Invariance Principle 35
2.5 Survival Copulas and Copula Symmetries 40
2.6 Measures of Association 45
2.6.1 Fallacies Related to the Correlation Coefficient 46
2.6.2 Rank Correlation Measures 51
2.6.3 Tail Dependence Coefficients 58
2.7 Rosenblatt Transform and Conditional Sampling 68
References 77

3 Classes and Families 81
3.1 Elliptical Distributions and Copulas 81
3.1.1 Elliptical Distributions 82
3.1.2 Elliptical Copulas 85
3.2 Archimedean Copulas 97
3.3 Extreme-Value Copulas 112
3.4 Selected Copula Transformations and Constructions 117
3.4.1 Rotated Copulas 117
3.4.2 Khoudraji's Device 120
3.4.3 Mixtures of Copulas 127
References 130

4 Estimation 133
4.1 Estimation Under a Parametric Assumption on the Copula 133
4.1.1 Parametrically Estimated Margins 134
4.1.2 Nonparametrically Estimated Margins 139
4.1.3 Estimators of Elliptical Copula Parameters 148
4.1.4 Other Semi-parametric Estimators 152
4.1.5 Estimation of Copula Models with Partly Fixed Parameters 153
4.2 Nonparametric Estimation of the Copula 157
4.2.1 The Empirical Copula 158
4.2.2 Under Extreme-Value Dependence 161
References 163

5 Graphical Diagnostics, Tests, and Model Selection 167
5.1 Basic Graphical Diagnostics 168
5.2 Hypothesis Tests 173
5.2.1 Tests of Independence 173
5.2.2 Tests of Exchangeability 176
5.2.3 A Test of Radial Symmetry 178
5.2.4 Tests of Extreme-Value Dependence 179
5.2.5 Goodness-of-Fit Tests 181
5.2.6 A Mixture of Graphical and Formal Goodness-of-Fit Tests 188
5.3 Model Selection 191
References 195

6 Ties, Time Series, and Regression 197
6.1 Ties 198
6.2 Selected Copula Tests and Models for Time Series 216
6.2.1 Tests of Stationarity 216
6.2.2 Tests of Serial Independence 226
6.2.3 Models for Multivariate Time Series Based on Conditional Copulas 230
6.3 Regression 238
References 252

A R and Package Versions 255

Index 259