
Contents

Preface	page ix
Acknowledgements	xi
1 Introduction to Symmetries	1
1.1 Symmetries of Planar Objects	1
1.2 Symmetries of the Simplest ODE	5
1.3 The Symmetry Condition for First-Order ODEs	8
1.4 Lie Symmetries Solve First-Order ODEs	11
2 Lie Symmetries of First-Order ODEs	15
2.1 The Action of Lie Symmetries on the Plane	15
2.2 Canonical Coordinates	22
2.3 How to Solve ODEs with Lie Symmetries	26
2.4 The Linearized Symmetry Condition	30
2.5 Symmetries and Standard Methods	34
2.6 The Infinitesimal Generator	38
3 How to Find Lie Point Symmetries of ODEs	43
3.1 The Symmetry Condition	43
3.2 The Determining Equations for Lie Point Symmetries	46
3.3 Linear ODEs	52
3.4 Justification of the Symmetry Condition	54
4 How to Use a One-Parameter Lie Group	58
4.1 Reduction of Order by Using Canonical Coordinates	58
4.2 Variational Symmetries	63
4.3 Invariant Solutions	68

5	Lie Symmetries with Several Parameters	74
5.1	Differential Invariants and Reduction of Order	74
5.2	The Lie Algebra of Point Symmetry Generators	79
5.3	Stepwise Integration of ODEs	89
6	Solution of ODEs with Multiparameter Lie Groups	93
6.1	The Basic Method: Exploiting Solvability	93
6.2	New Symmetries Obtained During Reduction	99
6.3	Integration of Third-Order ODEs with $\mathfrak{sl}(2)$	101
7	Techniques Based on First Integrals	108
7.1	First Integrals Derived from Symmetries	108
7.2	Contact Symmetries and Dynamical Symmetries	116
7.3	Integrating Factors	122
7.4	Systems of ODEs	128
8	How to Obtain Lie Point Symmetries of PDEs	136
8.1	Scalar PDEs with Two Dependent Variables	136
8.2	The Linearized Symmetry Condition for General PDEs	146
8.3	Finding Symmetries by Computer Algebra	149
9	Methods for Obtaining Exact Solutions of PDEs	155
9.1	Group-Invariant Solutions	155
9.2	New Solutions from Known Ones	162
9.3	Nonclassical Symmetries	166
10	Classification of Invariant Solutions	173
10.1	Equivalence of Invariant Solutions	173
10.2	How to Classify Symmetry Generators	176
10.3	Optimal Systems of Invariant Solutions	182
11	Discrete Symmetries	187
11.1	Some Uses of Discrete Symmetries	187
11.2	How to Obtain Discrete Symmetries from Lie Symmetries	188
11.3	Classification of Discrete Symmetries	191
11.4	Examples	195
	Hints and Partial Solutions to Some Exercises	201
	Bibliography	209
	Index	211