Contents

T

	instances and from Symmetries	
Pref	face	page ix
Ack	knowledgements	xi
1	Introduction to Symmetries	1
1.1	Symmetries of Planar Objects	1
1.2	Symmetries of the Simplest ODE	5
1.3	The Symmetry Condition for First-Order ODEs	8
1.4	Lie Symmetries Solve First-Order ODEs	11
2	Lie Symmetries of First-Order ODEs	15
2.1	The Action of Lie Symmetries on the Plane	15
2.2	Canonical Coordinates	22
2.3	How to Solve ODEs with Lie Symmetries	26
2.4	The Linearized Symmetry Condition	30
2.5	Symmetries and Standard Methods	34
2.6	The Infinitesimal Generator	38
3	How to Find Lie Point Symmetries of ODEs	43
3.1	The Symmetry Condition	43
3.2	The Determining Equations for Lie Point Symmetries	46
3.3	Linear ODEs	52
3.4	Justification of the Symmetry Condition	54
4	How to Use a One-Parameter Lie Group	58
4.1	Reduction of Order by Using Canonical Coordinates	58
4.2	Variational Symmetries	63
4.3	Invariant Solutions	68

Co	ntonts	
CO	nienis	

5	Lie Symmetries with Several Parameters	74
5.1	Differential Invariants and Reduction of Order	74
5.2	The Lie Algebra of Point Symmetry Generators	79
5.3	Stepwise Integration of ODEs	89
6	Solution of ODEs with Multiparameter Lie Groups	93
6.1	The Basic Method: Exploiting Solvability	93
6.2	New Symmetries Obtained During Reduction	99
6.3	Integration of Third-Order ODEs with $\mathfrak{sl}(2)$	101
7	Techniques Based on First Integrals	108
7.1	First Integrals Derived from Symmetries	108
7.2	Contact Symmetries and Dynamical Symmetries	116
7.3	Integrating Factors	122
7.4	Systems of ODEs	128
8	How to Obtain Lie Point Symmetries of PDEs	136
8.1	Scalar PDEs with Two Dependent Variables	136
8.2	The Linearized Symmetry Condition for General PDEs	146
8.3	Finding Symmetries by Computer Algebra	149
9	Methods for Obtaining Exact Solutions of PDEs	155
9 9.1	Methods for Obtaining Exact Solutions of PDEs Group-Invariant Solutions	155 155
9 9.1 9.2	Methods for Obtaining Exact Solutions of PDEs Group-Invariant Solutions New Solutions from Known Ones	155 155 162
9 9.1 9.2 9.3	Methods for Obtaining Exact Solutions of PDEs Group-Invariant Solutions New Solutions from Known Ones Nonclassical Symmetries	155 155 162 166
9 9.1 9.2 9.3 10	Methods for Obtaining Exact Solutions of PDEs Group-Invariant Solutions New Solutions from Known Ones Nonclassical Symmetries Classification of Invariant Solutions	155155162166173
 9 9.1 9.2 9.3 10 10.1 	Methods for Obtaining Exact Solutions of PDEs Group-Invariant Solutions New Solutions from Known Ones Nonclassical Symmetries Classification of Invariant Solutions Equivalence of Invariant Solutions	 155 155 162 166 173 173
9 9.1 9.2 9.3 10 10.1 10.2	Methods for Obtaining Exact Solutions of PDEs Group-Invariant Solutions New Solutions from Known Ones Nonclassical Symmetries Classification of Invariant Solutions Equivalence of Invariant Solutions How to Classify Symmetry Generators	 155 155 162 166 173 176
9 9.1 9.2 9.3 10 10.1 10.2 10.3	Methods for Obtaining Exact Solutions of PDEsGroup-Invariant SolutionsNew Solutions from Known OnesNonclassical SymmetriesClassification of Invariant SolutionsEquivalence of Invariant SolutionsHow to Classify Symmetry GeneratorsOptimal Systems of Invariant Solutions	 155 155 162 166 173 176 182
 9 9.1 9.2 9.3 10 10.1 10.2 10.3 11 	Methods for Obtaining Exact Solutions of PDEs Group-Invariant Solutions New Solutions from Known Ones Nonclassical Symmetries Classification of Invariant Solutions Equivalence of Invariant Solutions How to Classify Symmetry Generators Optimal Systems of Invariant Solutions	 155 155 162 166 173 173 176 182 187
 9 9.1 9.2 9.3 10 10.1 10.2 10.3 11 11.1 	Methods for Obtaining Exact Solutions of PDEsGroup-Invariant SolutionsNew Solutions from Known OnesNonclassical SymmetriesClassification of Invariant SolutionsEquivalence of Invariant SolutionsHow to Classify Symmetry GeneratorsOptimal Systems of Invariant SolutionsDiscrete SymmetriesSome Uses of Discrete Symmetries	 155 155 162 166 173 173 176 182 187 187
 9 9.1 9.2 9.3 10 10.1 10.2 10.3 11 11.1 11.2 	 Methods for Obtaining Exact Solutions of PDEs Group-Invariant Solutions New Solutions from Known Ones Nonclassical Symmetries Classification of Invariant Solutions Equivalence of Invariant Solutions How to Classify Symmetry Generators Optimal Systems of Invariant Solutions Some Uses of Discrete Symmetries How to Obtain Discrete Symmetries from Lie Symmetries 	 155 155 162 166 173 173 176 182 187 188
 9 9.1 9.2 9.3 10 10.1 10.2 10.3 11 11.1 11.2 11.3 	Methods for Obtaining Exact Solutions of PDEs Group-Invariant Solutions New Solutions from Known Ones Nonclassical Symmetries Classification of Invariant Solutions Equivalence of Invariant Solutions How to Classify Symmetry Generators Optimal Systems of Invariant Solutions Some Uses of Discrete Symmetries How to Obtain Discrete Symmetries from Lie Symmetries Classification of Discrete Symmetries	 155 155 162 166 173 173 176 182 187 187 188 191
 9 9.1 9.2 9.3 10 10.1 10.2 10.3 11 11.1 11.2 11.3 11.4 	Methods for Obtaining Exact Solutions of PDEsGroup-Invariant SolutionsNew Solutions from Known OnesNonclassical SymmetriesClassification of Invariant SolutionsEquivalence of Invariant SolutionsHow to Classify Symmetry GeneratorsOptimal Systems of Invariant SolutionsSome Uses of Discrete SymmetriesHow to Obtain Discrete Symmetries from Lie SymmetriesClassification of Discrete SymmetriesExamples	 155 155 162 166 173 173 176 182 187 188 191 195
 9 9.1 9.2 9.3 10 10.1 10.2 10.3 11 11.2 11.3 11.4 Hints 	Methods for Obtaining Exact Solutions of PDEsGroup-Invariant SolutionsNew Solutions from Known OnesNonclassical SymmetriesClassification of Invariant SolutionsEquivalence of Invariant SolutionsHow to Classify Symmetry GeneratorsOptimal Systems of Invariant SolutionsDiscrete SymmetriesHow to Obtain Discrete Symmetries from Lie SymmetriesHow to Obtain Discrete SymmetriesHow to Obtain Discrete SymmetriesHow to Obtain Discrete SymmetriesHow to Application of Discrete SymmetriesHo	 155 155 162 166 173 173 176 182 187 188 191 195 201
 9 9.1 9.2 9.3 10 10.1 10.2 10.3 11 11.1 11.2 11.3 11.4 Hints Biblic 	Methods for Obtaining Exact Solutions of PDEs Group-Invariant Solutions New Solutions from Known Ones Nonclassical Symmetries Classification of Invariant Solutions Equivalence of Invariant Solutions How to Classify Symmetry Generators Optimal Systems of Invariant Solutions Discrete Symmetries Some Uses of Discrete Symmetries How to Obtain Discrete Symmetries Classification of Discrete Symmetries Examples and Partial Solutions to Some Exercises	 155 155 162 166 173 173 176 182 187 188 191 195 201 209
 9 9.1 9.2 9.3 10 10.1 10.2 10.3 11 11.2 11.3 11.4 Hints Biblic L.1 	Methods for Obtaining Exact Solutions of PDEs Group-Invariant Solutions New Solutions from Known Ones Nonclassical Symmetries Classification of Invariant Solutions Equivalence of Invariant Solutions How to Classify Symmetry Generators Optimal Systems of Invariant Solutions Discrete Symmetries How to Obtain Discrete Symmetries from Lie Symmetries Classification of Discrete Symmetries Examples and Partial Solutions to Some Exercises	 155 155 162 166 173 173 176 182 187 188 191 195 201 209 211

viii