Contents

Preface v

I BACKGROUND 1

1 Introduction 3

2 Continuous One-Parameter Groups-I 7
2.1 Group Concept 7
2.2 Infinitesimal Transformation 10
2.3 Global Group Equations 13
2.4 Problems 17

3 Method of Characteristics 19
3.1 Theory 19
3.2 Examples 22
3.3 Problems 25

4 Continuous One-Parameter Groups-II 27
4.1 Invariance 27
4.2 The Once-Extended Group 33
4.3 Higher-Order Extended Groups 34
4.4 Problems 35

II ORDINARY DIFFERENTIAL EQUATIONS 37

5 First-Order ODEs 39
5.1 Invariance Under a One-Parameter Group 40
5.2 Canonical Coordinates 48
5.3 Special Procedures 53
5.4 Compendium 65
5.5 Examples 69
5.6 Problems 77

6 Higher-Order ODEs 81
6.1 Invariant Equations 82
6.2 Finding the Groups 88
6.3 System of First-Order ODEs 90
6.4 Compendium 94
6.5 Examples 95
6.6 Problems 105

7 Second-Order ODEs 109
7.1 Classification of Two-Parameter Groups 109
7.2 Invariance and Canonical Coordinates 113
7.3 Compendium 122
7.4 Examples 132
7.5 Problems 140

III Appendices 143

A Bibliography and References 145

B The Rotation Group 147

C Basic Relations 149

D Tables 153

E Answers to Selected Problems 213