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As the world's population crosses the six billion mark, the question, "How many people can Earth support and under what conditions?" has, at least as pressing as it ever was, become. It was posed it at the end of the eighteenth century in the pages of the *Encyclopédie de l'Population*.

It is a question about growing populations within existing economic systems and systems that are one of the main concerns of societies throughout history. What does the future of the Earth Support? is a recent book by J.E. Cohen (1995), which he derives from historical and scientific perspectives possible responses to the question. Historical "solutions" to the question of overpopulation have been based on two underlying assumptions: first, that under constant per-capita rates of population growth a population increases exponentially, that is, population "explosion" is observed; second, that resource limitations necessarily limit population to the magnitude of such an explosion. The usefulness and validity of these two views are naturally limited since the environment, often called the "carrying capacity" does not remain fixed. Per capita rates of population growth are not fixed by the conditions of changing environments. A limiting factor in the development of a society on both practical and theoretical terms: theory of population dynamics lies in the inability of theoreticians to provide models and frameworks with which to understand them. The environmental landscape in which we live is constantly changing, experiencing dramatic shifts due to technological innovations (such as nuclear power, climate change, famine, carbon emissions, and war) that periodically shake the world of what we think is possible. Cohen observes that population patterns are not "growth" rates depend on our scale of observation in both time and space. By now, Cohen, they are definitely not constant. For example, Cohen notes that in the fourteenth century repeated waves of Black Death, a form of bubonic plague, together with wars, heavy taxes, malnutrition, and poor and sometimes malicious governments, killed an estimated one third of the population living in India and China, and that the population of Meso-America fell by perhaps 80 or 90 percent during the sixteenth century (pp. 38–41). However, in spite of such sharp short-term decreases, world population size has actually grown steadily since prehistoric times, although not at a constant rate.