Sets and Extensions in the Twentieth Century
Volume 6 of the Handbook of the History of Logic

This volume of the *Handbook of the History of Logic* can be construed as the one focusing on *extensions*, collections of objects. It was in the mathematical development of set theory from Georg Cantor onwards that extensions became explicit through the mathematization of infinitary concepts and the further development of mathematics. The development of modern set theory is the subject of the first eight chapters, most of them set in the standard context of ZFC set theory and the last about its alternatives. Each chapter is long and detailed as befits the extent and richness of their wide-ranging subjects. The alternative to set theory as such for developing extensions has been, as first put forward by Bertrand Russell, to have explicit *types*. Types have come to the fore once again with the development of category theory and the emergence of computer science. The chapter on sets, types, and categories provides a sweeping view, and the chapter on categorical logic provides a tightly knit account of the "Montreal school". Finally, going full circle back to Russell, the last chapter of this volume discusses uses of Russell's *orders*, his original ramification of types, in modern logic and computer science.

Extensions and extensional logic are part and parcel of modern mathematics, and as such, of significant import for the philosophy of logic and of mathematics. This volume provides the first extended historical account of sets and, more broadly, extensions, and as such will be an invaluable resource for all those interested in the development of the very idea of collections of objects.

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