

Contents

Preface	IX
Introduction	1
1 Function Spaces, Linear Operators and Green's Functions	5
1.1 Function Spaces	5
1.2 Orthonormal System of Functions	7
1.3 Linear Operators	8
1.4 Eigenvalues and Eigenfunctions	11
1.5 The Fredholm Alternative	12
1.6 Self-adjoint Operators	15
1.7 Green's Functions for Differential Equations	16
1.8 Review of Complex Analysis	21
1.9 Review of Fourier Transform	28
2 Integral Equations and Green's Functions	33
2.1 Introduction to Integral Equations	33
2.2 Relationship of Integral Equations with Differential Equations and Green's Functions	39
2.3 Sturm-Liouville System	44
2.4 Green's Function for Time-Dependent Scattering Problem	48
2.5 Lippmann-Schwinger Equation	52
2.6 Problems for Chapter 2	57
3 Integral Equations of Volterra Type	63
3.1 Iterative Solution to Volterra Integral Equation of the Second Kind	63
3.2 Solvable cases of Volterra Integral Equation	66
3.3 Problems for Chapter 3	71
4 Integral Equations of the Fredholm Type	75
4.1 Iterative Solution to the Fredholm Integral Equation of the Second Kind	75
4.2 Resolvent Kernel	78
4.3 Pincherle-Goursat Kernel	81
4.4 Fredholm Theory for a Bounded Kernel	86
4.5 Solvable Example	93

4.6	Fredholm Integral Equation with a Translation Kernel	95
4.7	System of Fredholm Integral Equations of the Second Kind	100
4.8	Problems for Chapter 4	101
5	Hilbert-Schmidt Theory of Symmetric Kernel	109
5.1	Real and Symmetric Matrix	109
5.2	Real and Symmetric Kernel	111
5.3	Bounds on the Eigenvalues	122
5.4	Rayleigh Quotient	126
5.5	Completeness of Sturm-Liouville Eigenfunctions	129
5.6	Generalization of Hilbert-Schmidt Theory	131
5.7	Generalization of Sturm-Liouville System	138
5.8	Problems for Chapter 5	144
6	Singular Integral Equations of Cauchy Type	149
6.1	Hilbert Problem	149
6.2	Cauchy Integral Equation of the First Kind	153
6.3	Cauchy Integral Equation of the Second Kind	157
6.4	Carleman Integral Equation	161
6.5	Dispersion Relations	166
6.6	Problems for Chapter 6	173
7	Wiener-Hopf Method and Wiener-Hopf Integral Equation	177
7.1	The Wiener-Hopf Method for Partial Differential Equations	177
7.2	Homogeneous Wiener-Hopf Integral Equation of the Second Kind	191
7.3	General Decomposition Problem	207
7.4	Inhomogeneous Wiener-Hopf Integral Equation of the Second Kind	216
7.5	Toeplitz Matrix and Wiener-Hopf Sum Equation	227
7.6	Wiener-Hopf Integral Equation of the First Kind and Dual Integral Equations	235
7.7	Problems for Chapter 7	239
8	Nonlinear Integral Equations	249
8.1	Nonlinear Integral Equation of Volterra type	249
8.2	Nonlinear Integral Equation of Fredholm Type	253
8.3	Nonlinear Integral Equation of Hammerstein type	257
8.4	Problems for Chapter 8	259
9	Calculus of Variations: Fundamentals	263
9.1	Historical Background	263
9.2	Examples	267
9.3	Euler Equation	267
9.4	Generalization of the Basic Problems	272
9.5	More Examples	276
9.6	Differential Equations, Integral Equations, and Extremization of Integrals	278
9.7	The Second Variation	283

9.8	Weierstrass-Erdmann Corner Relation	297
9.9	Problems for Chapter 9	300
10	Calculus of Variations: Applications	303
10.1	Feynman's Action Principle in Quantum Mechanics	303
10.2	Feynman's Variational Principle in Quantum Statistical Mechanics	308
10.3	Schwinger-Dyson Equation in Quantum Field Theory	312
10.4	Schwinger-Dyson Equation in Quantum Statistical Mechanics	329
10.5	Weyl's Gauge Principle	339
10.6	Problems for Chapter 10	356
	Bibliography	365
	Index	373