

Series Prefacev
Preface to the Third Editionvii
Preface to the Second Editionix
Preface to the First Editionxi
0 Basic Concepts1
0.1 Weak Formulation of Boundary Value Problems1
0.2 Ritz-Galerkin Approximation3
0.3 Error Estimates4
0.4 Piecewise Polynomial Spaces - The Finite Element Method7
0.5 Relationship to Difference Methods9
0.6 Computer Implementation of Finite Element Methods10
0.7 Local Estimates12
0.8 Adaptive Approximation13
0.9 Weighted Norm Estimates15
0.x Exercises19
1 Sobolev Spaces23
1.1 Review of Lebesgue Integration Theory23
1.2 Generalized (Weak) Derivatives26
1.3 Sobolev Norms and Associated Spaces29
1.4 Inclusion Relations and Sobolev's Inequality32
1.5 Review of Chapter 035
1.6 Trace Theorems36
1.7 Negative Norms and Duality40
1.x Exercises42

2 Variational Formulation of Elliptic Boundary Value Problems	49
2.1 Inner-Product Spaces	49
2.2 Hilbert Spaces	51
2.3 Projections onto Subspaces	52
2.4 Riesz Representation Theorem	55
2.5 Formulation of Symmetric Variational Problems	56
2.6 Formulation of Nonsymmetric Variational Problems	59
2.7 The Lax-Milgram Theorem	60
2.8 Estimates for General Finite Element Approximation	64
2.9 Higher-dimensional Examples	65
2.x Exercises	66
3 The Construction of a Finite Element Space	69
3.1 The Finite Element	69
3.2 Triangular Finite Elements	71
The Lagrange Element	72
The Hermite Element	75
The Argyris Element	76
3.3 The Interpolant	77
3.4 Equivalence of Elements	81
3.5 Rectangular Elements	85
Tensor Product Elements	85
The Serendipity Element	86
3.6 Higher-dimensional Elements	87
3.7 Exotic Elements	89
3.x Exercises	90
4 Polynomial Approximation Theory in Sobolev Spaces	93
4.1 Averaged Taylor Polynomials	93
4.2 Error Representation	96
4.3 Bounds for Riesz Potentials	100
4.4 Bounds for the Interpolation Error	105
4.5 Inverse Estimates	111
4.6 Tensor-product Polynomial Approximation	113
4.7 Isoparametric Polynomial Approximation	118
4.8 Interpolation of Non-smooth Functions	119
4.9 A Discrete Sobolev Inequality	123
4.x Exercises	125

5 n-Dimensional Variational Problems	129
5.1 Variational Formulation of Poisson's Equation	129
5.2 Variational Formulation of the Pure Neumann Problem.	132
5.3 Coercivity of the Variational Problem.	134
5.4 Variational Approximation of Poisson's Equation	136
5.5 Elliptic Regularity Estimates	138
5.6 General Second-Order Elliptic Operators.	141
5.7 Variational Approximation of General Elliptic Problems.	144
5.8 Negative-Norm Estimates	146
5.9 The Plate-Bending Biharmonic Problem.	148
5.x Exercises	151
6 Finite Element Multigrid Methods	155
6.1 A Model Problem.	155
6.2 Mesh-Dependent Norms.	157
6.3 The Multigrid Algorithm.	159
6.4 Approximation Property.	161
6.5 W-cycle Convergence for the k^{th} Level Iteration	162
6.6 V-cycle Convergence for the k^{th} Level Iteration	165
6.7 Full Multigrid Convergence Analysis and Work Estimates——	170
6.x Exercises	172
7 Additive Schwarz Preconditioned	175
7.1 Abstract Additive Schwarz Framework.	175
7.2 The Hierarchical Basis Preconditioner.	179
7.3 The BPX Preconditioner.	183
7.4 The Two-level Additive Schwarz Preconditioner.	185
7.5 Nonoverlapping Domain Decomposition Methods.	191
7.6 The BPS Preconditioner.	197
7.7 The Neumann-Neumann Preconditioner.	201
7.8 The BDDC Preconditioner.	205
7.x Exercises	210
8 Max-norm Estimates	215
8.1 Main Theorem.	215
8.2 Reduction to Weighted Estimates.	218
8.3 Proof of Lemma 8.2.6.	220
8.4 Proofs of Lemmas 8.3.7 and 8.3.11.	224
8.5 L^p Estimates (Regular Coefficients)	229

8.6 L^p Estimates (Irregular Coefficients)	231
8.7 A Nonlinear Example	235
8.x Exercises	238
9 Adaptive Meshes	241
9.1 A priori Estimates	242
9.2 Error Estimators	244
9.3 Local Error Estimates	247
9.4 Estimators for Linear Forms and Other Norms	249
9.5 A Convergent Adaptive Algorithm	253
9.6 Conditioning of Finite Element Equations	261
9.7 Bounds on the Condition Number	264
9.8 Applications to the Conjugate-Gradient Method	266
9.x Exercises	267
10 Variational Crimes	271
10.1 Departure from the Framework	
10.2 Finite Elements with Interpolated Boundary Conditions	274
10.3 Nonconforming Finite Elements	281
10.4 Isoparametric Finite Elements	286
10.5 Discontinuous Finite Elements	289
10.6 Poincare-Friedrichs Inequalities for Piecewise $W^{p,1}$ Functions	296
10.x Exercises	303
11 Applications to Planar Elasticity	311
11.1 The Boundary Value Problems	311
11.2 Weak Formulation and Horn's Inequality	313
11.3 Finite Element Approximation and Locking	320
11.4 A Robust Method for the Pure Displacement Problem	323
11.x Exercises	327
12 Mixed Methods	331
12.1 Examples of Mixed Variational Formulations	331
12.2 Abstract Mixed Formulation	333
12.3 Discrete Mixed Formulation	336
12.4 Convergence Results for Velocity Approximation	338
12.5 The Discrete Inf-Sup Condition	341
12.6 Verification of the Inf-Sup Condition	347
12.x Exercises	353

13 Iterative Techniques for Mixed Methods	355
13.1 Iterated Penalty Method	355
13.2 Stopping Criteria	359
13.3 Augmented Lagrangian Method	361
13.4 Application to the Navier-Stokes Equations	363
13.5 Computational Examples	366
13.x Exercises	369
14 Applications of Operator-Interpolation Theory	371
14.1 The Real Method of Interpolation	371
14.2 Real Interpolation of Sobolev Spaces	373
14.3 Finite Element Convergence Estimates	376
14.4 The Simultaneous Approximation Theorem	379
14.5 Precise Characterizations of Regularity	379
14.x Exercises	380
References	383
Index	393