

Contents

1	Probability	1
1.1	Fluctuations and Precision Limits	2
1.2	A History of Probabilistic Thinking	6
1.3	Interpretations of Probability	11
1.4	Sets and Sample Spaces	16
1.5	Probability Measure on Countable Sample Spaces	20
1.5.1	Probability Measure	21
1.5.2	Probability Weights	24
1.6	Discrete Random Variables and Distributions	27
1.6.1	Distributions and Expectation Values	27
1.6.2	Random Variables and Continuity	29
1.6.3	Discrete Probability Distributions	34
1.6.4	Conditional Probabilities and Independence	38
1.7	* Probability Measure on Uncountable Sample Spaces	44
1.7.1	* Existence of Non-measurable Sets	46
1.7.2	* Borel σ -Algebra and Lebesgue Measure	49
1.8	Limits and Integrals	55
1.8.1	Limits of Series of Random Variables	55
1.8.2	Riemann and Stieltjes Integration	59
1.8.3	Lebesgue Integration	63
1.9	Continuous Random Variables and Distributions	70
1.9.1	Densities and Distributions	71
1.9.2	Expectation Values and Variances	76
1.9.3	Continuous Variables and Independence	77
1.9.4	Probabilities of Discrete and Continuous Variables	78
2	Distributions, Moments, and Statistics	83
2.1	Expectation Values and Higher Moments	83
2.1.1	First and Second Moments	84
2.1.2	Higher Moments	91
2.1.3	* Information Entropy	95

2.2	Generating Functions	101
2.2.1	Probability Generating Functions.....	101
2.2.2	Moment Generating Functions	103
2.2.3	Characteristic Functions	105
2.3	Common Probability Distributions.....	107
2.3.1	The Poisson Distribution.....	109
2.3.2	The Binomial Distribution	111
2.3.3	The Normal Distribution	115
2.3.4	Multivariate Normal Distributions.....	120
2.4	Regularities for Large Numbers	124
2.4.1	Binomial and Normal Distributions	125
2.4.2	Central Limit Theorem	130
2.4.3	Law of Large Numbers.....	133
2.4.4	Law of the Iterated Logarithm	135
2.5	Further Probability Distributions	137
2.5.1	The Log-Normal Distribution.....	137
2.5.2	The χ^2 -Distribution	140
2.5.3	Student's t-Distribution	143
2.5.4	The Exponential and the Geometric Distribution.....	147
2.5.5	The Pareto Distribution	151
2.5.6	The Logistic Distribution	154
2.5.7	The Cauchy–Lorentz Distribution.....	156
2.5.8	The Lévy Distribution	159
2.5.9	The Stable Distribution	161
2.5.10	Bimodal Distributions	166
2.6	Mathematical Statistics	168
2.6.1	Sample Moments	169
2.6.2	Pearson's Chi-Squared Test.....	173
2.6.3	Fisher's Exact Test	180
2.6.4	The Maximum Likelihood Method.....	182
2.6.5	Bayesian Inference	190
3	Stochastic Processes	199
3.1	Modeling Stochastic Processes	203
3.1.1	Trajectories and Processes	203
3.1.2	Notation for Probabilistic Processes.....	208
3.1.3	Memory in Stochastic Processes.....	209
3.1.4	Stationarity	214
3.1.5	Continuity in Stochastic Processes	216
3.1.6	Autocorrelation Functions and Spectra.....	220
3.2	Chapman–Kolmogorov Forward Equations	224
3.2.1	Differential Chapman–Kolmogorov Forward Equation.....	225
3.2.2	Examples of Stochastic Processes	235
3.2.3	Master Equations	260

3.2.4	Continuous Time Random Walks	273
3.2.5	Lévy Processes and Anomalous Diffusion	284
3.3	Chapman–Kolmogorov Backward Equations	303
3.3.1	Differential Chapman–Kolmogorov Backward Equation ...	305
3.3.2	Backward Master Equations	307
3.3.3	Backward Poisson Process	310
3.3.4	Boundaries and Mean First Passage Times	313
3.4	Stochastic Differential Equations	319
3.4.1	Mathematics of Stochastic Differential Equations	321
3.4.2	Stochastic Integrals	323
3.4.3	Integration of Stochastic Differential Equations	337
4	Applications in Chemistry	347
4.1	A Glance at Chemical Reaction Kinetics	350
4.1.1	Elementary Steps of Chemical Reactions	351
4.1.2	Michaelis–Menten Kinetics	358
4.1.3	Reaction Network Theory	372
4.1.4	Theory of Reaction Rate Parameters	388
4.1.5	Empirical Rate Parameters	407
4.2	Stochasticity in Chemical Reactions	415
4.2.1	Sampling of Trajectories	416
4.2.2	The Chemical Master Equation	418
4.2.3	Stochastic Chemical Reaction Networks	425
4.2.4	The Chemical Langevin Equation	432
4.3	Examples of Chemical Reactions	435
4.3.1	The Flow Reactor	436
4.3.2	Monomolecular Chemical Reactions	441
4.3.3	Bimolecular Chemical Reactions	450
4.3.4	Laplace Transform of Master Equations	459
4.3.5	Autocatalytic Reaction	477
4.3.6	Stochastic Enzyme Kinetics	485
4.4	Fluctuations and Single Molecule Investigations	490
4.4.1	Single Molecule Enzymology	491
4.4.2	Fluorescence Correlation Spectroscopy	500
4.5	Scaling and Size Expansions	509
4.5.1	Kramers–Moyal Expansion	509
4.5.2	Small Noise Expansion	512
4.5.3	Size Expansion of the Master Equation	514
4.5.4	From Master to Fokker–Planck Equations	521
4.6	Numerical Simulation of Chemical Master Equations	526
4.6.1	Basic Assumptions	527
4.6.2	Tau-Leaping and Higher-Level Approaches	531
4.6.3	The Simulation Algorithm	533
4.6.4	Examples of Simulations	542

5 Applications in Biology	569
5.1 Autocatalysis and Growth	572
5.1.1 Autocatalysis in Closed Systems	572
5.1.2 Autocatalysis in Open Systems	575
5.1.3 Unlimited Growth	580
5.1.4 Logistic Equation and Selection	583
5.2 Stochastic Models in Biology	585
5.2.1 Master Equations and Growth Processes	585
5.2.2 Birth-and-Death Processes	589
5.2.3 Fokker–Planck Equation and Neutral Evolution	605
5.2.4 Logistic Birth-and-Death and Epidemiology	611
5.2.5 Branching Processes	631
5.3 Stochastic Models of Evolution	649
5.3.1 The Wright–Fisher and the Moran Process	651
5.3.2 Master Equation of the Moran Process	658
5.3.3 Models of Mutation	665
5.4 Coalescent Theory and Phylogenetic Reconstruction	673
Notation	679
References	683
Author Index	707
Index	711