

# Quantum Mechanics

Concepts and Applications

Nouredine Zettili  
Jacksonville State University, USA

*Quantum Mechanics: Concepts and Applications* provides a clear, balanced introduction aimed at students taking an introductory course in the subject. Written with the students' background and ability in mind, the book takes an innovative approach to quantum mechanics. The text successfully combines essential theory with many practical applications, illustrated by numerous examples and solved problems. Carefully structured, the text opens with the experimental basis of quantum mechanics before covering the mathematical tools, such as linear spaces, operator algebra, matrix mechanics and eigenvalue problems. Subsequent chapters cover the formal foundations of quantum mechanics, the exact solutions of the Schrödinger equation for one and three dimensional potentials, time independent and time-dependent approximation methods and then finally, the theory of scattering.

Assuming no prior knowledge of the subject, this richly illustrated text includes many worked examples and numerous problems with step by step solutions designed to help the reader master the machinery of quantum mechanics.

*Quantum Mechanics: Concepts and Applications;*

- Provides a comprehensive introduction to quantum mechanics, combining both a theoretical and practical approach.
- Includes over 65 solved examples integrated throughout the text and each chapter concludes with an extensive collection of fully solved multipart problems.
- Offers an in-depth treatment of the practical mathematical tools of quantum mechanics.
- Devotes an entire section to the numerical solution of the one-dimensional Schrödinger equation, (including code)

This introductory text is aimed at all students taking a first course on quantum mechanics. It promises to become an invaluable tool not only for graduate students preparing for the preliminary and qualifying examinations but for teachers as well.

Nouredine Zettili received his Ph.D. in 1986 from MIT and is currently Professor of Physics at Jacksonville State University, USA. His research interests include nuclear theory, the many-body problem, quantum mechanics and mathematical physics. He has also published two booklets designed to help students improve their study skills.

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