

# ANALYSIS OF MESSY DATA

## VOLUME 1

### DESIGNED EXPERIMENTS

SECOND EDITION

A bestseller for nearly 25 years, **Analysis of Messy Data, Volume 1: Designed Experiments** helps you analyze the kinds of data sets encountered in the real world. Written by two long-time researchers and professors, this second edition has been fully updated to reflect the many developments that have occurred since the original publication.

#### New to the Second Edition

- Several modern suggestions for multiple comparison procedures
- Additional examples of split-plot designs and repeated measures designs
- The use of SAS-GLM to analyze an effects model
- The use of SAS-MIXED and JMP to analyze data in random effects experiments, mixed model experiments, and repeated measures experiments

The book explores various techniques for multiple comparison procedures, random effects models, mixed models, split-plot experiments, and repeated measures designs. The authors implement the techniques using several statistical software packages and emphasize the distinction between design structure and the structure of treatments. They introduce each topic with examples, follow up with a theoretical discussion, and conclude with a case study. Bringing a classic work up to date, this edition will continue to show you how to effectively analyze real-world, nonstandard data sets.

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