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As an undergraduate student, I found it challenging to find a text for the course that contains the important ideas of statistical mechanics in a pedagogical and clear manner, to include mathematical detail in a way that is useful for undergraduates and beginning graduate students, while also being affordable. This necessarily leads to a limited scope. Unlike many existing texts on statistical physics, I do not try to cover thermal physics and statistical mechanics in the same book, nor do I try to be comprehensive in covering both undergraduate and graduate statistical mechanics in the same volume. My goal in this age book is one that will provide students and instructors with the essential ideas of statistical physics in a way that will prove useful to all students. For students continuing to graduate studies in physics and related topics I aim to give a comprehensive coverage of ideas that are needed in graduate school, and for students who do not continue to graduate studies I aim to give a solid background in statistical physics that will allow them to apply these ideas in other contexts.

The book is based on lecture notes for a one-semester (13 weeks, 3 hours of lectures/week) undergraduate course on statistical physics that I have delivered three times at Simon Fraser University (SFU). I have assumed that the reader of the book has already taken (or is taking at the same time) a course in thermal physics, so I do not elaborate greatly on thermodynamic quantities, although I have included a brief primer on thermal physics in Appendix B, so that the reader can look up ideas as needed. I also assume that the reader has taken at least one quantum mechanics course, so they are familiar with the solution of quantum-mechanical problems like the particle in a box and the simple harmonic oscillator. My target audience for this book is primarily undergraduate students learning statistical mechanics for the first time. However, I also believe that it will be of interest to graduate students wanting a clear reference for statistical mechanics that provides details of calculations and clarification of concepts, and to instructors looking for an affordable, well-structured text or reference for statistical physics courses.