

Donald Yau

Involutive Category Theory

This monograph introduces involutive categories and involutive operads, featuring applications to the GNS construction and algebraic quantum field theory. The author adopts an accessible approach for readers seeking an overview of involutive category theory, from the basics to cutting-edge applications. Additionally, the author's own recent advances in the area are featured, never having appeared previously in the literature.

The opening chapters offer an introduction to basic category theory, ideal for readers new to the area. Chapters three through five feature previously unpublished results on coherence and strictification of involutive categories and involutive monoidal categories, showcasing the author's state-of-the-art research. Chapters on coherence of involutive symmetric monoidal categories, and categorical GNS construction follow. The last chapter covers involutive operads and lays important coherence foundations for applications to algebraic quantum field theory.

With detailed explanations and exercises throughout, *Involutive Category Theory* is suitable for graduate seminars and independent study. Mathematicians and mathematical physicists who use involutive objects will also find this a valuable reference.

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