Contents

1 Introduction 1

Part I Physics Preliminaries

2 Relativistic Point Particle 9
- 2.1 BV Action for the Point Particle 9
- 2.2 Scattering Matrix and Minimal Model 16
- 2.3 Summary, Comments, and Remarks Towards Part II 22

3 String Theory 27
- 3.1 Closed Strings 27
- 3.2 Interactions 33
- 3.3 Decomposition of the Moduli Space 39
- 3.4 Measure, Vertices, and BV Action 41
- 3.5 Algebraic Structure 45
- 3.6 Coalgebra Description 47
- 3.7 Uniqueness and Background Independence 49
- 3.8 Summary, Comments, and Remarks Towards Part II 50

4 Open and Closed Strings 55
- 4.1 World-Sheets with Boundaries 55
- 4.2 Open String 58
- 4.3 Summary, Comments, and Remarks Towards Part II 62

5 Open-Closed BV Equation 65
- 5.1 Open-Closed BV Action 65
- 5.2 Quantum Open-Closed Homotopy Algebra 67
- 5.3 Summary, Comment, and Remarks Towards Part II 73

Part II Mathematical Interpretation

6 Operads 95
- 6.1 Cyclic Operads 95
- 6.2 Non-Σ Cyclic Operads 126
- 6.3 Operad Algebras 131

6.4 Modular Operads 133
6.5 Odd Modular Operads 148
References 157

7 Feynman Transform of a Modular Operad 159
7.1 Modules and Derivations 159
7.2 Feynman Transform 170
References 184

8 Structures Relevant to Physics 185
8.1 BV Algebras and the Master Equation 185
8.2 Loop Homotopy Algebras 200
8.3 IBL_∞-algebras 207
8.4 Comments and Remarks Related to Part I 214
References 217

Index 219