

Contents

1	Introduction: Signals and Transforms	1
1.1	Continuous-Time Periodic Signals	1
1.1.1	One-Dimensional Periodic Signals	1
1.1.2	Two-Dimensional Periodic Signals	3
1.2	Periodic Discrete-Time Periodic Signals	4
1.2.1	One-Dimensional Periodic Discrete-Time Signals	5
1.2.2	Two-Dimensional Periodic Discrete-Time Signals	7
	References	8
2	Introduction: Periodic Filters and Filter Banks	9
2.1	Periodic Filters	9
2.1.1	Definition of Periodic Filters	9
2.1.2	Multirate p-Filtering	10
2.2	Periodic Filter Banks	12
2.2.1	Filter Banks	12
2.2.2	Characterization of p-Filter Banks	13
	Reference	14
3	Mixed Circular Convolutions and Zak Transforms	15
3.1	Periodic Discrete-Continuous Circular Convolution and Zak Transform	15
3.2	A Leading Example: Periodic Polynomial Splines	18
3.2.1	Periodic B-Splines	19
3.2.2	Spaces of Periodic Splines	21
3.3	Periodic Discrete-Discrete Convolution and Zak Transform	24
3.4	A Leading Example: Periodic Discrete Splines	28
3.4.1	Non-periodic Discrete B-Splines	28
3.4.2	Periodic Discrete B-Splines	28
3.4.3	Spaces of Periodic Discrete Splines	29
	References	33

4	Periodic Polynomial Splines	35
4.1	Spline Harmonic Analysis (SHA) in the Space of Periodic Polynomial Splines.	35
4.1.1	Orthogonal Periodic Exponential Splines.	35
4.1.2	Normalized Periodic Exponential Splines	39
4.1.3	Representation of Periodic Splines by Exponential Splines Bases.	40
4.1.4	Generators of Periodic Splines Spaces.	44
4.1.5	Remarks on Orthonormal Bases of Splines Spaces and SHA Spectra	53
4.2	SHA in Two-Dimensional Spline Spaces.	56
	References	58
5	Polynomial Smoothing Splines	59
5.1	One-Dimensional Smoothing Splines	59
5.1.1	Solution of the Unconstrained Minimization Problem.	60
5.1.2	Solution of the Constrained Minimization Problem.	62
5.1.3	Generators of Smoothing Splines	64
5.2	Two-Dimensional Smoothing Splines	66
	References	68
6	Calculation of Splines Values by Subdivision	69
6.1	Interpolatory Subdivision.	70
6.2	Binary Subdivision for Periodic Splines	71
6.2.1	Spline Spaces on Different Dyadic Resolution Scales	71
6.2.2	Insertion Rule	73
6.2.3	Periodic Spline Filters for Binary Subdivision	74
6.2.4	Computation of Periodic Splines at Dyadic Rational Points.	76
6.3	Ternary Periodic Spline Subdivision	77
6.3.1	Super-Resolution Spline Spaces (Triadic Scale)	77
6.3.2	Insertion Rule	79
6.3.3	Periodic Spline Filters for Ternary Subdivision	79
6.3.4	Computation of Periodic Splines at Triadic Rational Points.	82
6.3.5	Practical Implementation	84
6.4	Two-dimensional Spline Subdivision	85
6.5	Upsampling Signals and Images.	86
6.5.1	Upsampling Discrete-Time signals	86
6.5.2	Upsampling Digital Images	89
	References	95

7	Spline Algorithms for Deconvolution and Inversion of Heat Equation	97
7.1	Spline Algorithms for Deconvolution	97
7.1.1	Solution of One-Dimensional Convolution Equations	98
7.1.2	Solution of Two-Dimensional Convolution Equations	106
7.2	Inversion of Heat Equation	113
7.2.1	Inversion of One-Dimensional Heat Equation	117
7.2.2	Inversion of Two-Dimensional Heat Equation	121
	References	131
8	Periodic Spline Wavelets and Wavelet Packets	133
8.1	Spline Spaces of Different Resolution Scales	134
8.2	Two-Scale Relations	136
8.2.1	Two-Scale Relations in Spline Spaces	136
8.2.2	Orthogonal Complements to Spline Spaces	137
8.2.3	Refined Split of the Spline Space into Orthogonal Subspaces	139
8.3	Spline Wavelet Packet Transforms	142
8.3.1	Transform Matrices	142
8.3.2	One-Level Transforms	143
8.3.3	Transforms to Coarser Levels	151
8.4	Spline Wavelet Packets	157
8.4.1	Definition of Spline Wavelet Packets	157
8.4.2	The SHA Spectra of Spline Wavelet Packets	158
8.4.3	Wavelet Packets Bases	161
8.4.4	Best Basis	165
8.4.5	Reconstruction of a Spline from Wavelet Packet Bases	169
8.5	2D Wavelet Packets	170
8.5.1	2D Wavelet Packets Bases	170
8.5.2	2D Wavelet Packets Transforms	172
8.5.3	Best Basis	180
	References	182
9	Discrete-Time Periodic Wavelet Packets	183
9.1	One-Dimensional Periodic Discrete-Time Wavelet Packets	183
9.1.1	Summary of One-Level Wavelet Transform of a Spline	183
9.1.2	One-Level Wavelet Transform of a Signal	186
9.1.3	Multilevel Wavelet Packet Transforms of a Signal	191
9.1.4	Bases of Discrete-Time Wavelet Packets	194

9.2	Two-Dimensional Discrete Periodic Wavelet Packets	204
9.2.1	One-Level 2D Wavelet Transform	204
9.2.2	Multilevel Wavelet Packet Transform	206
9.2.3	2D Wavelet Bases	209
10	Deconvolution by Regularized Matching Pursuit	215
10.1	Outline of the Matching Pursuit Scheme	216
10.1.1	Conventional MP	216
10.1.2	Outline of the Regularized MP for Deconvolution	217
10.2	Description of the RMP for Deconvolution	218
10.2.1	Dictionaries for RMP	219
10.2.2	Modelling Noise	220
10.2.3	Oblique Projection	221
10.2.4	Stopping Threshold and Reduction of Dictionaries	225
10.2.5	Pursuit	227
10.2.6	Computational Scheme	228
10.3	RMP for Inversion of Heat Equation	231
10.3.1	Dictionaries for Heat Equation RMP	231
10.3.2	Computational Scheme	233
10.4	Examples	236
10.4.1	Conclusions	245
	References	246
11	Block-Based Inversion of the Heat Equations	247
11.1	One-Dimensional Block-Based Heat Equation Inversion	248
11.1.1	Preliminaries	248
11.1.2	Partial Solution of the Inversion Problem in the Subspace ${}^{2r}\mathcal{W}_{m,l}$	250
11.1.3	Approximate Inversion of Heat Equation in the Space ${}^{2r}\mathcal{S}$	255
11.2	2D Block-Based Heat Equation Inversion	258
11.2.1	Preliminaries	258
11.2.2	Partial Solution of the Inversion Problem in the Subspace ${}^{2r}\mathcal{W}_{m,l,\tilde{l}}$	259
11.2.3	Scheme for the Approximate Inversion of the 2D Heat Equation	264
11.3	Numerical Examples	267
	References	273
12	Hydro-Acoustic Target Detection	275
12.1	Outline of the Problem	275
12.2	Structure of the Recorded Data	277
12.3	Formulation of the Approach	278

12.4	Description of the Algorithm and its Implementation	281
12.4.1	Derivation of the Classes Signatures	281
12.4.2	Training the Classifiers	285
12.4.3	Identification of an Acoustic Signal	287
12.5	Examples	288
12.5.1	Structure of Presented Figures	289
12.5.2	Display of Results	290
	References	294
13	Periodic Discrete Splines	295
13.1	One-dimensional Discrete Spline Harmonic Analysis	295
13.1.1	Discrete Periodic Splines Spaces	295
13.1.2	Exponential Discrete Periodic Splines	297
13.1.3	Normalized Exponential Discrete Periodic Splines	302
13.1.4	Representation of Discrete Periodic Splines by Exponential Splines Basis	304
13.1.5	Generators of Periodic Discrete Splines' Spaces	307
13.1.6	Smoothing Periodic Discrete Splines	314
13.2	Two-dimensional Discrete Spline Harmonic Analysis	319
13.2.1	Bases in Two-dimensional Discrete Spline Spaces	319
13.2.2	Outline of 2D DSHA Relations	321
13.2.3	Explicit Computation of a 2D Periodic Discrete Spline	322
13.2.4	2D Smoothing Periodic Discrete Splines	324
	References	330
14	Discrete Periodic Spline Wavelets and Wavelet Packets	331
14.1	Two-Scale Relations	331
14.1.1	Orthogonal Complements of Discrete Spline Spaces	332
14.1.2	Refined Split of the Signal Space $\Pi[N]$ into Orthogonal Subspaces	333
14.2	Discrete Spline Wavelet Packet Transforms	335
14.2.1	Transform Matrices	335
14.2.2	One-Level Transforms	336
14.2.3	Multilevel Wavelet Packet Transforms	340
14.3	Discrete Spline Wavelet Packets	344
14.3.1	The DFT Spectra of Spline Wavelet Packets	346
14.3.2	Discrete Splines Wavelet Packets Bases	349
14.4	2D Wavelet Packets	352
14.4.1	2D Discrete Spline Wavelet Packets Bases	354
14.4.2	2D Wavelet Packets Transforms	356

14.5	Deconvolution and Denoising by Regularized Matching Pursuit	362
14.5.1	Outline of the RMP for Deconvolution	362
14.5.2	Computational Scheme	365
14.5.3	Numerical Examples	368
	References	376
15	Biorthogonal Wavelet Transforms	377
15.1	Two-Channel Filter Banks	378
15.1.1	Matrix Expression of p-Filter Banks	378
15.1.2	Biorthogonal Bases Generated by PR p-Filter Banks	380
15.1.3	Multilevel Wavelet Transforms	382
15.1.4	Implementation of Multi-Level Wavelet Transforms	386
15.2	Compactly Supported Biorthogonal Wavelets	388
15.2.1	Biorthogonal Low-Pass FIR p-Filters	389
15.2.2	Examples	390
15.3	Restoration of Sampled Polynomials and Discrete Vanishing Moments	394
15.3.1	Restoration of Sampled Polynomials	394
15.3.2	Discrete Vanishing Moments	395
15.3.3	Examples	396
	References	397
16	Biorthogonal Wavelet Transforms Originating from Splines	399
16.1	Lifting Scheme of Wavelet Transforms	399
16.1.1	Lifting Steps	399
16.1.2	Filter Banks	400
16.2	Prediction Filters Derived from Polynomial Splines	403
16.2.1	Periodic Interpolating Splines	404
16.2.2	Prediction p-filters	405
16.2.3	Approximation Properties of Spline p-filters	406
16.2.4	Perfect Reconstruction p-filter Banks	407
16.2.5	Examples of p-filters Derived from Polynomial Splines	408
16.3	Prediction Filters Derived from Discrete Splines	412
16.3.1	Summary for the Discrete Splines of Span 2	412
16.3.2	Filter Banks	413
	References	416

17	Wavelet Frames Generated by Spline Based p-Filter Banks	417
17.1	Oversampled PR Filter Banks and Frames	418
17.1.1	Oversampled p-Filter Banks with Downsampling Factor of 2	418
17.1.2	Frames in the Space of Periodic Signals	420
17.2	Design of Interpolating Three-Channel p-Filter Banks Generating Frames	426
17.2.1	Interpolating p-Filter Banks for Frame Generation	426
17.2.2	Tight and Semi-Tight Frames	429
17.2.3	Interpolating Three-Channel p-Filter Banks Using Spline Filters	430
17.3	Design of Four-Channel p-Filter Banks for Frames Generation	438
17.3.1	Four-Channel Perfect Reconstruction p-Filter Banks	438
17.3.2	Low-Pass p-Filters	443
17.4	Four-Channel p-Filter Banks Using Spline Filters	445
17.4.1	Examples of p-Filter Banks With p-FIR p-Filters	447
17.4.2	Four-Channel p-Filter Banks With IIR p-Filters	455
	References	463
18	Application of Periodic Frames to Image Restoration	465
18.1	Outline of the Restoration Scheme	465
18.2	Numerical Examples	466
18.2.1	Restoration Experiments with the “Window” Image	467
18.2.2	Restoration Experiments with the “Barbara” Image	470
18.2.3	Restoration Experiments with the “Boats” Image	473
18.2.4	Restoration Experiments with the “Lena” Image	475
18.2.5	Restoration Experiments with the “Fingerprint” Image	475
18.3	Comments on the Experiments	476
	References	478
	Appendix: Guide to SplineSoftP	479
	Glossary	489
	Index	493

$$f(t) = \sum_{n \in \mathbb{Z}} f_n(t - nT)$$