

Contents

| | | |
|----------|--|----|
| 1 | Introduction: Signals and Transforms | 1 |
| 1.1 | Continuous-Time Decaying Signals | 1 |
| 1.1.1 | One-Dimensional Signals | 1 |
| 1.1.2 | Two-Dimensional Signals | 4 |
| 1.2 | Discrete-Time Signals | 5 |
| 1.2.1 | One-Dimensional Discrete-Time Signals | 5 |
| 1.2.2 | Two-Dimensional Discrete Signals | 7 |
| | References | 8 |
| 2 | Introduction: Digital Filters and Filter Banks | 9 |
| 2.1 | Filtering Decaying Signals | 9 |
| 2.1.1 | Filters | 9 |
| 2.1.2 | Filter Banks | 13 |
| 2.1.3 | Polyphase Representation | 13 |
| 2.1.4 | Interpolating Filters | 17 |
| 2.2 | Bases and Frames Generated by Filter Banks | 20 |
| 2.2.1 | Examples of Filter Banks Implementation | 21 |
| 2.2.2 | Implementation of Causal—Anticausal IIR Filters with a Rational Transfer Function | 25 |
| 2.3 | Discrete-Time Butterworth Filters | 27 |
| | References | 30 |
| 3 | Mixed Convolutions and Zak Transforms | 31 |
| 3.1 | Mixed Discrete-Continuous Convolution and Zak Transform | 31 |
| 3.1.1 | Mixed Discrete-Continuous Convolution | 31 |
| 3.1.2 | Zak Transform of Continuous-Time Signals | 32 |
| 3.2 | A Leading Example: Polynomial Splines | 35 |
| 3.2.1 | B-Splines | 35 |
| 3.2.2 | Spline Spaces | 37 |

| | | |
|----------|---|------------|
| 3.3 | Discrete-Discrete Convolution and Zak Transform | 40 |
| 3.3.1 | A Leading Example: Discrete Splines | 43 |
| | References. | 49 |
| 4 | Non-periodic Polynomial Splines | 51 |
| 4.1 | Integral Representation of Splines | 51 |
| 4.1.1 | Decaying Polynomial Splines | 51 |
| 4.1.2 | Zak Transform of B-Spline (Exponential Spline). | 52 |
| 4.1.3 | Integral Representation of Splines | 54 |
| 4.2 | Generators of Spline Spaces | 58 |
| 4.2.1 | Examples of the Spline Space Generators | 59 |
| 4.3 | Polynomial Interpolating Splines. | 68 |
| 4.3.1 | Example: Cubic Interpolating Spline | 69 |
| 4.3.2 | Exponential Decay of Generators of Spline Spaces | 71 |
| 4.3.3 | Minimal Norm Property of Even-Order Splines | 73 |
| 4.4 | Polynomial Smoothing Splines (Global). | 74 |
| 4.4.1 | Explicit Expression of Smoothing Splines | 75 |
| 4.4.2 | Computation of Smoothing Splines | 78 |
| 4.4.3 | Smoothing Generators for the Spline Spaces | 79 |
| 4.4.4 | Examples. | 82 |
| | References. | 85 |
| 5 | Quasi-interpolating and Smoothing Local Splines. | 87 |
| 5.1 | Moments of B-Splines. | 87 |
| 5.2 | Local Splines | 90 |
| 5.2.1 | Simplest (Variation-Diminishing) Splines. | 90 |
| 5.2.2 | Quasi-interpolating Splines. | 91 |
| 5.2.3 | Examples. | 105 |
| 5.3 | Approximation Properties of Splines | 108 |
| | References. | 113 |
| 6 | Cubic Local Splines on Non-uniform Grid | 115 |
| 6.1 | Preliminaries: Divided Differences and Interpolating Polynomials | 115 |
| 6.2 | B-Splines. | 117 |
| 6.3 | Cubic Splines. | 117 |
| 6.3.1 | Simplest Cubic Splines | 118 |
| 6.3.2 | Quasi-interpolating Cubic Splines | 119 |
| 6.4 | Examples. | 123 |
| | References. | 125 |
| 7 | Splines Computation by Subdivision | 127 |
| 7.1 | Interpolatory Subdivision for Non-periodic Splines | 127 |
| 7.2 | Binary Subdivision for Non-periodic Splines | 128 |

| | | |
|-----------|---|------------|
| 7.2.1 | Spline Filters for Binary Subdivision | 129 |
| 7.2.2 | Splines Computation at Dyadic Rational Points | 131 |
| 7.2.3 | Practical Implementation | 132 |
| 7.3 | Ternary Subdivision | 135 |
| 7.3.1 | Spline Filters for Ternary Subdivision | 135 |
| 7.3.2 | Splines Computation at Triadic Rational Points | 137 |
| 7.3.3 | Practical Implementation | 140 |
| 7.4 | Upsampling Examples | 142 |
| | References | 144 |
| 8 | Polynomial Spline-Wavelets | 145 |
| 8.1 | Two-scale Relations | 147 |
| 8.1.1 | Two-scale Relations for Exponential Splines | 147 |
| 8.1.2 | Exponential Wavelets | 149 |
| 8.2 | Spline-Wavelet Transforms | 151 |
| 8.2.1 | One Step of the Spline Wavelet Transforms | 151 |
| 8.2.2 | Multiscale Spline Wavelet Transforms | 155 |
| 8.3 | Generators of the Wavelet Space | 156 |
| 8.3.1 | B-Wavelets | 157 |
| 8.3.2 | Generators and Dual Generators | 162 |
| 8.3.3 | Examples of the Wavelet-space Generators | 164 |
| | References | 169 |
| 9 | Non-periodic Discrete Splines | 171 |
| 9.1 | Discrete Splines' Spaces | 171 |
| 9.2 | Integral Representation of Discrete Splines | 173 |
| 9.2.1 | Exponential Discrete Splines | 173 |
| 9.2.2 | Characteristic Functions of the Discrete Splines' Spaces | 175 |
| 9.2.3 | Implications of the Integral Representation of Discrete Splines | 177 |
| 9.2.4 | Computation of Discrete Splines | 178 |
| 9.3 | Generators of Discrete Splines' Spaces | 181 |
| 9.3.1 | Generators and Their Duals | 181 |
| 9.3.2 | Examples of Generators | 183 |
| | References | 188 |
| 10 | Non-periodic Discrete-Spline Wavelets | 189 |
| 10.1 | Discrete Splines' Spaces | 189 |
| 10.2 | Discrete Exponential Wavelets | 191 |
| 10.3 | Generators of the Discrete Spline Wavelet Spaces | 193 |
| 10.3.1 | Discrete B-Wavelets | 194 |
| 10.3.2 | Generators and Dual Generators of Discrete Spline Wavelet Spaces | 196 |

| | | |
|-----------|---|------------|
| 10.4 | Wavelet Transforms of Discrete Splines | 202 |
| 10.4.1 | Matrix Representation of the Two-Scale Relations | 202 |
| 10.4.2 | Transform of Splines' Coordinates | 203 |
| 10.5 | Discrete-Spline Wavelet Transform of Signals | 206 |
| 10.5.1 | One Step of Discrete-Spline Wavelet Transform | 206 |
| 10.5.2 | Multiscale Signal's Transform | 211 |
| 10.5.3 | Examples | 211 |
| | References | 214 |
| 11 | Biorthogonal Wavelet Transforms | 215 |
| 11.1 | Two-Channel Filter Banks | 216 |
| 11.1.1 | Matrix Expression of Filter Banks | 216 |
| 11.1.2 | Biorthogonal Bases Generated by PR Filter Banks | 218 |
| 11.1.3 | Multilevel Discrete-Time Wavelet Transforms | 221 |
| 11.2 | Compactly Supported Biorthogonal Wavelets | 228 |
| 11.2.1 | Design of the Biorthogonal Filter Bank | 228 |
| | References | 237 |
| 12 | Biorthogonal Wavelet Transforms Originating from Splines | 239 |
| 12.1 | Lifting Scheme of Wavelet Transforms | 239 |
| 12.1.1 | Primal Scheme | 239 |
| 12.1.2 | Dual Scheme | 240 |
| 12.1.3 | Filter Banks | 241 |
| 12.2 | Filter Banks Originating from Polynomial Splines | 245 |
| 12.2.1 | Prediction Filters Derived from Polynomial Splines | 245 |
| 12.2.2 | Filter Banks | 247 |
| 12.2.3 | Examples of Filters Originating from Splines | 252 |
| 12.3 | Filter Banks Originating from Discrete Splines | 261 |
| 12.3.1 | Summary for the Discrete Splines of Span 2 | 261 |
| 12.3.2 | Prediction Filters | 263 |
| | References | 273 |
| 13 | Data Compression Using Wavelet and Local Cosine Transforms | 275 |
| 13.1 | Spatial and Spectral Meaning of Wavelet Transform Coefficients | 275 |
| 13.2 | SPIHT Coding Scheme | 279 |
| 13.3 | Local Cosine Transform for Data Compression | 281 |
| 13.3.1 | Local Cosine Transform (LCT) | 281 |
| 13.3.2 | A Hybrid Algorithm for Data Compression | 282 |
| 13.4 | Numerical Examples | 286 |
| 13.4.1 | Seismic Compression | 287 |
| 13.4.2 | Fingerprints | 289 |

| | | |
|-------------|--|------------|
| 13.4.3 | Compression of Multimedia Images | 291 |
| 13.4.4 | Conclusions | 295 |
| References. | | 296 |
| 14 | Wavelet Frames Generated by Perfect Reconstruction | |
| | Filter Banks | 299 |
| 14.1 | Oversampled Filter Banks and Frames. | 300 |
| 14.1.1 | Matrix Expression of Filter Banks with the Downsampling Factor 2. | 300 |
| 14.1.2 | Frames Generated by Filter Banks | 303 |
| 14.2 | Design of Three-Channel Filter Banks Which Generate Frames. | 308 |
| 14.2.1 | Interpolating Filter Banks for Frame Generation | 308 |
| 14.2.2 | Frames Derived from Triangular Factorization | 310 |
| 14.2.3 | Design of Frames Using Spline Filters. | 312 |
| 14.2.4 | 2D Frame Transforms | 329 |
| 14.3 | Design of Four-Channel Filter Banks Which Generate Frames. | 331 |
| 14.3.1 | Four-Channel Oversampled Filter Banks | 331 |
| 14.3.2 | Low-Pass Filters | 336 |
| 14.4 | Four-Channel Filter Banks Using Spline Filters | 340 |
| 14.4.1 | Summary of the Filter Bank Design Scheme | 340 |
| 14.4.2 | Outline of the Frame Transforms' Implementation | 341 |
| 14.4.3 | Examples of Filter Banks with FIR Filters | 343 |
| 14.4.4 | Four-Channel Filter Banks with IIR Filters. | 350 |
| References. | | 360 |
| 15 | Biorthogonal Multiwavelets Originated from Hermite Splines | 363 |
| 15.1 | Preliminaries | 364 |
| 15.1.1 | Cubic Hermite Splines. | 364 |
| 15.1.2 | Multifilters. | 365 |
| 15.2 | Lifting Scheme of Wavelet Transform of Vector-Signals | 368 |
| 15.3 | Multifilter Banks. | 370 |
| 15.3.1 | Structure of Multifilter Banks. | 370 |
| 15.3.2 | Approximation Properties of Multifilters | 372 |
| 15.4 | Lifting Algorithms for Pre/Post-processing Phases. | 374 |
| 15.4.1 | An Orthogonal Scheme of Third Approximation Order (Haar Algorithm). | 375 |
| 15.4.2 | Schemes of the Fifth Approximation Order | 376 |
| 15.5 | Bases for the Space of Discrete-Time Signals. | 380 |
| 15.5.1 | Bases of Zero Level | 380 |
| 15.5.2 | Bases of the First Level. | 381 |

| | | |
|-----------|---|------------|
| 15.6 | Extension of the Multiwavelet Transforms to Coarser Levels | 388 |
| 15.7 | Two-Dimensional Multiwavelet Transforms | 391 |
| | References. | 392 |
| 16 | Multiwavelet Frames Originated From Hermite Splines | 393 |
| 16.1 | Oversampled Multifilter Banks | 394 |
| 16.1.1 | Three-Channel Multifilter Banks | 394 |
| 16.1.2 | Bases of Zero Level | 395 |
| 16.1.3 | Analysis Filter Banks of the First Level | 396 |
| 16.1.4 | Synthesis Filter Banks of the First Level | 397 |
| 16.2 | Multiwavelet Frames | 398 |
| 16.2.1 | Signal's Expansion Over the First-Level Multi-frame | 398 |
| 16.2.2 | Extension of the Multi-frame Transforms to Coarser Levels | 399 |
| 16.3 | Design of Multifilter Banks Generating Frames | 401 |
| 16.3.1 | Design Scheme. | 401 |
| 16.3.2 | Transfer Functions of the Multifilter Banks | 402 |
| 16.3.3 | Example: Framelets Originating From the Haar Pre-Post-processing Scheme ($j = 0$): | 404 |
| | References. | 407 |
| | Appendix A: Guide to SplineSoftN | 409 |
| | Glossary | 421 |
| | Index | 423 |