Contents

1 Introduction: Periodic Signals and Filters 1
1.1 Periodic Discrete-Time Signals 1
1.1.1 One-Dimensional Periodic Discrete-Time Signals 1
1.1.2 Two-Dimensional Periodic Discrete-Time Signals 4
1.2 Periodic Filters 5
1.2.1 Definition of Periodic Filters 5
1.2.2 Multirate p-Filtering 6
1.3 Periodic Filter Banks 8
1.3.1 Modulation Matrices of p-Filter Banks 8
1.3.2 Polyphase Matrices of p-Filter Banks 9
References 10

2 Periodic Polynomial Splines 11
2.1 Periodic B-Splines and Spline Spaces 11
2.2 Exponential Splines and Characteristic Sequences 13
2.3 Spline Harmonic Analysis (SHA) in the Space of Periodic Polynomial Splines 14
2.3.1 Orthogonal Periodic Exponential Splines 14
2.3.2 Representation of Periodic Splines by Exponential Spline Bases 15
2.4 SHA in Two-Dimensional Spline Spaces 19
References 20

3 Periodic Discrete and Discrete-Time Splines 21
3.1 Periodic Discrete Splines of Span 2 21
3.1.1 B-Splines and Discrete Spline Spaces 21
3.1.2 Exponential Discrete Splines and Characteristic Sequences 23

3.1.3 Interpolating and Quasi-interpolating Discrete Splines . . . 25
3.1.4 Orthonormal Periodic Discrete Splines . . . 27
3.2 Two-Dimensional Discrete Splines of Span 2 . . . 30
3.3 Periodic Discrete-Time Splines . . . 32
3.3.1 B-Splines and Discrete-Time Spline Spaces . . . 32
3.3.2 Exponential Discrete-Time Splines and Characteristic Sequences . . . 35
3.3.3 Orthonormal Periodic Discrete-Time Splines . . . 37
3.3.4 Interpolating Discrete-Time Splines . . . 39
References . . . 40

4 Periodic Orthogonal Wavelets and Wavelet Packets . . . 41
4.1 Discrete-Spline Wavelets of First Decomposition Level . . . 41
4.1.1 Orthogonal Complement for Subspace ${}^{2r}\mathscr{S}^0_{[1]}$ . . . 41
4.1.2 Orthogonal Complement for Subspace ${}^{p}\bar{\mathscr{S}}^0_{[1]}$ . . . 44
4.1.3 One-Level Wavelet Transform of a Signal . . . 45
4.2 Two-Level Discrete-Spline Wavelet and Wavelet Packets Transforms . . . 48
4.2.1 P-Filter Banks for the Second Decomposition Level . . . 48
4.2.2 Two-Level Wavelet Transforms . . . 50
4.2.3 Two-Level Wavelet Packet Transforms . . . 55
4.2.4 Selection of Wavelet-Packet Basis . . . 59
4.3 Extension of Wavelet Packet Transforms to Lower Decomposition Levels . . . 61
4.3.1 Discrete-Spline Wavelet Packets . . . 61
4.3.2 Discrete-Time-Spline Wavelet Packets . . . 63
4.4 Best Basis and Signal's Restoration from Wavelet Packet Bases . . . 64
4.5 Illustrations . . . 66
4.6 Waveband Spectra . . . 68
References . . . 72

5 Two-Dimensional Orthogonal Wavelets and Wavelet Packets . . . 73
5.1 Two-Dimensional Discrete-Spline and Discrete-Time-Spline Wavelets . . . 73
5.1.1 Wavelets of the First Decomposition Level . . . 73
5.1.2 Multi-level 2D Wavelet Transforms . . . 76
5.2 Two-Dimensional Discrete-Spline Wavelet Packets . . . 78
5.2.1 Direct Wavelet Packet Transforms . . . 78
5.2.2 Best Basis and Restoration of 2D Signals From Wavelet Packet Bases . . . 80

6 Local Splines on Non-uniform Grid 83
6.1 Preliminaries 83
6.2 Local Cubic Splines 86
6.2.1 Simplest Cubic Splines 87
6.2.2 Quasi-interpolating Cubic Splines 88
6.2.3 Two-Dimensional Cubic Splines 97
6.3 Local Quadratic Splines 100
6.3.1 Design of Quadratic Quasi-interpolating Splines 100
6.3.2 Extension of Splines to Boundaries and Extrapolation 105
6.3.3 Quadratic Splines on Uniform Grids 107
6.4 Examples: Restoration of *Chirp* Function 110
6.4.1 Cubic Splines 110
6.4.2 Quadratic Splines 111
References 113

7 Spline-Based Wavelet Transforms 115
7.1 Discrete Lifting Wavelet Transforms 115
7.1.1 One-Level Transform 115
7.1.2 1-D Multilevel Wavelet Transform 117
7.2 Spline-Based Prediction and Updating Operators 117
7.2.1 Prediction and Updating Operators – Cubic Splines . . . 118
7.2.2 Prediction and Updating Operators – Quadratic Splines 129
References 132

8 Biorthogonal Wavelet Transforms Originating from Discrete and Discrete-Time Splines 133
8.1 Filter Banks Related to Lifting Scheme of Wavelet Transforms 133
8.1.1 Lifting Steps 133
8.1.2 Filter Banks 134
8.2 Prediction and Updating p-Filters Derived from Discrete Splines 136
8.2.1 Summary for the Discrete Splines of Span 2 136
8.2.2 Filter Banks 136
8.3 Prediction and Updating p-Filters Derived from Discrete-Time Splines 140
8.3.1 Preliminaries 140
8.3.2 Filter Banks 142
8.4 Discrete Vanishing Moments 145
References 147

9 Wavelet Frames Generated by Spline-Based p-Filter Banks 149
9.1 Oversampled PR Filter Banks and Frames 150
9.1.1 Oversampled p-Filter Banks with Downsampling Factor 2 150
9.1.2 Frames in the Space of Periodic Signals 151
9.2 Design of p-Filter Banks for Frames Generation 156
9.2.1 Four-Channel Perfect Reconstruction p-Filter Banks . . . 156
9.2.2 Symmetric Tight Frames with Increased Redundancy 161
9.3 Examples of PR p-Filter Banks Using Spline Filters 162
9.3.1 Examples of Six-Channel p-Filter Banks Generating Tight Frames 162
9.3.2 Examples of Eight-Channel FIR p-Filter Banks Generating Tight Frames 166
9.3.3 Four-Channel p-Filter Banks with IIR p-Filters Derived from Discrete Splines 170
9.3.4 Example of Image Restoration Using Tight Frames 172
References 175

10 Snapshot Spectral Imaging 177
10.1 Background 177
10.2 Spectral Imaging with a Dispersive Diffuser 178
10.2.1 Optical Scheme 178
10.3 Compressed Sensing Model for Spectral Imaging 180
10.3.1 Discrete Mathematical Model 181
10.4 Solution of the Reconstruction Problem 183
10.4.1 Rationale Behind the Method 183
10.4.2 Outline of Algorithm 184
10.5 Experimental Optical Arrangement and Calibration 185
10.6 Optical Experiments for Spectral Imaging 187
10.7 Comments 194
References 195

11 Delineation of Malignant Skin Tumors by Hyperspectral Imaging 197
11.1 Background 197
11.2 Method 198
11.2.1 Imaging Device 198
11.2.2 Outline of the Processing Scheme 199
11.2.3 Selection of Training Data 201
11.2.4 Preprocessing the Training Data 202
11.2.5 Selection of Characteristic Features 204
11.2.6 Dimension Reduction 207

11.2.7 Decision Units 209
11.2.8 Identification Phase 210
11.3 Experimental Results 211
11.4 Comments 213
11.5 Appendix: Diffusion Framework 213
11.5.1 Diffusion Maps 213
11.5.2 Geometric Harmonics 215
11.6 Appendix: Outline of the CART Algorithm 216
References 218

12 Acoustic Detection of Moving Vehicles 219
12.1 Introduction 219
12.2 Structure of the Recorded Acoustics Signals 220
12.3 Formulation of the Approach 222
12.3.1 Outline of the Approach 223
12.4 Description of the Algorithm and Its Implementation 224
12.4.1 The Algorithm 224
12.4.2 Choice of the Analyzing Waveforms 225
12.4.3 Training the Algorithm 226
12.4.4 Identification of an Acoustic Signal 228
12.5 Experimental Results 230
12.5.1 Detection Experiments 231
12.5.2 Examples 233
12.6 Appendix: Random Search for a Near Optimal Footprint (RSNOFP) Scheme 236
References 239

13 Detection of Incipient Bearing Fault in a Slowly Rotating Machine Using Spline Wavelet Packets 241
13.1 Preliminaries 241
13.1.1 Background 241
13.1.2 Outline of the Methodology 243
13.1.3 Time-Domain Meaning of Wavelet Packet Transform Coefficients 244
13.2 WPT-Based Methodology for the Fault Detection 245
13.2.1 Structure of the Recorded Data 246
13.2.2 Outline of the Research Methodology 248
13.3 Experimental Results 249
13.3.1 Experiments with *Bearing1* 249
13.3.2 Experiments with *Bearing2* 255
13.4 Comments 259
13.5 Appendix: Envelope 260
References 262

Appendix A: Guide to SB_Soft1 263

Glossary 275

Bibliography 279

Index 285