

VOLUME III

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For a system composed of identical particles, the particle numbering used in Chapter 10 (the last chapter of Volume II [3]), does not really have much physical significance. In fact, when the particle number gets larger than a few units, applying the symmetrization postulate to numbered particles often leads to complex calculations. For