

Contents

I. Fundamental concepts

1. Linear algebra	1
2. Underlying spaces	3
3. Localization and genericity	4
4. The blanket principle	5
5. Remarks to modules	6
6. Differential forms	7
7. Two generalizations	8
8. Vector fields	9
9. The Lie bracket	10
10. The exterior differential	10
11. The effect of a mapping	11
12. The Lie derivative intuitively	13
13. Infinitesimal transformations	14
14. The Lie derivative again	15
15. A digest of vector fields	16
Comments	18

II. Around the Frobenius theorem

1. Flatness and complete integrability	19
2. Classical Frobenius theorem	20
3. Adjoint variables	20
4. The Pfaff–Darboux normal shapes	24
5. Derived modules	25
6. Groups of transformations	25
7. On infinitesimal transformations	27
8. Groups of substitutions	27
9. Theorem	28
10. Approximation property for groups	30
11. Generalized Frobenius theorem	30
12. Other generalizations and perspectives	31
Comments	32

III. Ordinary differential equations

1. From Monge systems to diffieties	33
2. From diffieties to Monge systems	35
3. The Cartan filtrations	36
4. Standard filtrations	38
5. Simple remarks	39
6. Morphisms of diffieties	41
7. Elementary approach to subdiffieties	42

8. Elementary approach to factordiffieties	42
9. The contact forms	43
10. Automorphisms of contact forms	46
11. Continuation: explicit calculations	48
12. Contact forms and parameters	51
13. The Sturm-Liouville equation	51
14. Continuation: solution with quadratures	54
15. Infinitesimal symmetries	55
16. Miscellanery of symmetries	57
Comments	59
IV. The Monge problem	
1. An outlook and the strategy of solution	61
2. Factordiffieties through Cartan filtrations	62
3. The comparison lemma	63
4. The cross-sections method	66
5. Application: a reduction principle	67
6. Solvability with one function	68
7. Continuation: another approach	69
8. Solvability with one function and constants	71
9. Solvability with two functions	72
10. Continuation: the semi-structural lemma	73
11. Termination: the structural lemma	76
12. Solvability with one function and one quadrature	77
13. Continuation: a particular example	82
14. Continuation: yet more particular examples	84
15. Identification of diffieties	85
16. Identification of contact diffieties	86
17. Identification of quadratures	87
Comments	89
V. Formal calculus of variations	
1. Stationary points	91
2. The Lagrange problem	92
3. The moving boundary problem	94
4. Generalized Poincaré-Cartan forms	95
5. Definitions, simple results, future tasks	97
6. A general strategy for the inverse problem	100
7. First order non-degenerate integrals	102
8. Two remarks on the resolving system	105
9. An alternative approach	106
10. Continuation: a few particular subcases	107
11. A degenerate variational problem	109
12. The underdetermined Euler-Lagrange system	113
13. Notes to higher order variational problems	114
14. Two examples of constrained variational integrals	116

Comments	118
VI. On commutative algebra	
1. Notation and terminology	119
2. The Koszul complex	121
3. The Hilbert resolution	123
4. Associated ideals and regularity of sequences	126
5. The Chevalley-Krull-Samuel theorem	129
6. Remarks on the multiplicity	129
7. The technique of generic sequences	130
8. The homologies once more	132
9. The regularity of sequences and involutiveness	133
10. Introduction of characteristics	138
11. Continuation: four remarks	140
12. On alternative homologies	141
Comments	144
VII. Partial differential equations	
1. Diffieties, morphisms, infinitesimal symmetries	145
2. Links between diffieties and differential equations	146
3. Evolution diffieties	148
4. Links to commutative algebra	149
5. The multiplicity result	153
6. The absolute Cauchy characteristics	155
7. Lemma	156
8. Toward the integrability theorem	157
9. Pseudodifferential operators and fractions	159
10. Proof of the main theorem	162
11. Integrability of characteristics	163
12. The Cartan filtration	164
13. The second acyclicity existence theorem	167
14. The involutiveness existence theorem	170
15. Subdiffieties of contact diffiety	170
16. Standard filtrations	176
17. On symmetries and infinitesimal symmetries	177
18. Remarks to linearization	182
19. Remarks to underdetermined diffieties	183
Comments	183
VIII. The Lie-Cartan pseudogroups	
1. The classical equivalence problem	185
2. Illustrative examples	190
3. Slightly generalized diffieties	195
4. Groupieties	196
5. Some properties of Maurer-Cartan forms	197
6. Axioms for Maurer-Cartan forms	200

7. The uniqueness of Maurer-Cartan forms	200
8. Dual axioms	201
9. Essential invariants and adapted coordinates	202
10. The existence of adapted coordinates	204
11. Simple examples	207
12. Subgroupieties	212
13. Normal subgroupieties	213
14. Intrinsic filtrations and composition series	215
15. Pseudogroups of diffeomorphisms	216
16. Examples of subgroupieties	220
17. Homogeneous spaces and related concepts	225
18. The left and the right cosets	227
19. Missed topics	229
Comments	229

IX. The multiple variational integrals

1. A general class of variational problems	231
2. Few notes to the classical approach	232
3. The case of contact diffieties again	235
4. To the Poincaré lemma	242
5. The variational bicomplex	243
6. Towards the spectral sequences	246
7. One-dimensional variational bicomplex	247
8. Continuation: yet the inverse problem	249
9. Two-dimensional variational bicomplex	250
10. Continuation: few examples	255
11. Links to commutative algebra	258
12. Few notes to general bicomplex	260
13. Comeback of calculus of variations	262
Comments	264

X. Appendices

1. The Lie's main theorems	265
2. A comparison of Lie groups and pseudogroups	273
3. The spectral sequences	274
4. A report on diffieties with symmetries	277
5. To lower order differential equations	281
6. Concluding examples	286
Comments	290

Index

References