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presented here by using recent methods and results in algebra, topology, differential geometry, functional analysis, theory of functional equations and linear differential equations of the second order, and by introducing several original methods, global solutions of problems which were previously studied only locally by Kummer, Brianchi, Laguerre, Poincaré, Halphen, Lie, Stäckel and others are provided. The structure of global transformations is described by algebraic means (theory of categories, Brauer and Thompson groups); a new geometrical approach is introduced that leads to global canonical forms (in contrast to the local Laguerre-Poincaré or Halphen forms) and is suitable for the application of Cartan's moving frame of reference method. The results contain also a criterion of global equivalence of two linear differential equations which is in general effective for $n \geq 3$ and new global invariants of linear differential equations with respect to global transformations.

The theory also provides effective tools for solving some open problems in particular concerning the distribution of zeros of solutions. The theory of functional equations plays an important role in studying the asymptotic behavior of solutions. The book also contains applications to other fields of mathematics, especially to differential geometry and functional equations. Some related results and further possible areas of research are mentioned at the end of the book.