Contents

List of notation ix

1 Preliminaries 1
- 1.1 Elements of Linear Algebra . . . 1
- 1.2 Normed Linear Spaces . . . 23

2 Properties of Linear and Nonlinear Operators 55
- 2.1 Linear Operators . . . 55
- 2.2 Compact Operators . . . 76
- 2.3 Contraction Principle . . . 91

3 Abstract Integral and Differential Calculus 103
- 3.1 Integration of Vector Functions . . . 103
- 3.2 Differential Calculus in Normed Linear Spaces . . . 110

4 Local Properties of Differentiable Mappings 129
- 4.1 Inverse Function Theorem . . . 129
- 4.2 Implicit Function Theorem . . . 136
- 4.3 Local Structure of Differentiable Maps, Bifurcations . . . 145

5 Topological Methods 171
- 5.1 Brouwer and Schauder Fixed Point Theorems . . . 171
- 5.2 Topological Degree . . . 183
- 5.3 Theory of Monotone Operators . . . 208
- 5.4 Supersolutions, Subsolutions, Monotone Iterations . . . 220

6 Variational Methods 231
- 6.1 Local Extrema . . . 231
- 6.2 Global Extrema . . . 245
- 6.3 Relative Extrema and Lagrange Multipliers . . . 260
- 6.4 Mountain Pass Theorem . . . 274
- 6.5 Saddle Point Theorem . . . 284

7 Boundary Value Problems for Partial Differential Equations 293
7.1 Classical Solution, Functional Setting 293
7.2 Classical Solution, Applications 297
7.3 Weak Solutions, Functional Setting 299
7.4 Weak Solutions, Application of Fixed Point Theorems 305
7.5 Weak Solutions, Application of the Degree Theory 312
7.6 Weak Solutions, Application of Theory of Monotone Operators 319
7.7 Weak Solutions, Application of Variational Methods 323

Index 335

Bibliography 349