

CONTENTS

Preface	9
1 Sets	13
1.1 Naive set theory	13
1.1.1 Set descriptions	14
1.1.2 Set operations	15
1.2 Axiomatic set theory	18
1.2.1 Paradoxes of the naive set theory	18
1.2.2 Sets and classes	19
1.2.3 Alternative set theory	20
1.3 Mappings	21
1.4 Cardinality of sets	24
1.5 Fuzzy sets	28
2 Relations, graphs, operations, algebras and structures	33
2.1 Relations	33
2.1.1 A relation with the generally different domains	34
2.1.2 Relations on a set	40
2.2 Binary relations, priorities and orders	41
2.2.1 Attributes of the binary relation	41
2.2.2 Equivalence	43
2.2.3 The relation of preference and order	44
2.2.4 The methods to describe relations	48
2.2.5 The partial dimension order	50
2.2.6 Preferences with non-differentiable relations	51

2.3	Graphs	54
2.3.1	Directed and not directed graphs	54
2.3.2	Some of the elementary properties of graphs	57
2.3.3	Diagrams of graphs, graph isomorphism	58
2.3.4	The valuated graphs	60
2.3.5	Walks, trails and paths	61
2.3.6	Some special graphs	63
2.3.7	The representation of graph by data for computation	66
2.4	Operations, algebras and theories	71
2.4.1	Operations	71
2.4.2	Algebras and theories	73
2.4.3	Structures	77
3	Information, data, codes	80
3.1	The concept of information	80
3.1.1	The principals of Shannon information theory	80
3.1.2	The principles of semantic information theory	83
3.2	Information, data, and coding	87
3.2.1	Data and their interpretation	87
3.2.2	Data redundancy and data protection	89
3.3	Data encryption and cryptography	91
3.3.1	Monoalphabetical ciphering	94
3.3.2	Polyalphabetical ciphering	94
3.3.3	Asymmetrical ciphering	95
4	Formal languages and grammars	98
4.1	The alphabet, word and language	98
4.1.1	The alphabet	99
4.1.2	Words	99
4.1.3	Language	101
4.2	Grammars	101
4.3	The Chomsky grammar hierarchy	105
4.3.1	The type 0 grammar	105
4.3.2	The type 1 grammar	106
4.3.3	The type 2 grammar	107
4.3.4	The type 3 grammar	107
4.4	The derivation tree	109

5	Formal computation models	116
5.1	The finite automaton and their modifications	117
5.1.1	The Mealy automaton	125
5.1.2	The Moore automaton	125
5.1.3	The non-deterministic finite automaton	127
5.1.4	The Nerode theorem	130
5.2	Regular expressions and languages	132
5.3	Pushdown automaton	137
5.3.1	The deterministic pushdown automaton	139
5.3.2	The non-deterministic pushdown automaton . .	141
5.4	The Turing machine	143
5.4.1	Decision making and recognizing languages by the Turing machine	147
5.5	The computability theory	150
5.5.1	Recursive and partial recursive functions	151
5.6	RASP machines	153
5.7	Petri nets	154
6	Computational complexity	158
6.1	Numerical stability of an algorithm	158
6.2	Time and space complexity: A practical and theoretical perspective	160
6.2.1	Asymptotic estimation of function growth. Symbols: O , o and Θ	160
6.2.2	The typical classes of computational complexity of algorithms	163
6.2.3	A theoretical view of complexity	166
6.2.4	A practical view of complexity	167
6.2.5	Pessimistic and average complexity	168
6.2.6	An example how an algorithm adjustment can influence the time complexity	169
6.3	P-difficult, NP-difficult, and NP-complete problems . .	175
6.3.1	Deterministic and non-deterministic complexity of problems	177
7	Examples of algorithms analysis	188
7.1	The divide and conquer principle	188
7.2	Searching and insertion	195
7.3	Ordering	200

7.3.1	Tag sort algorithms	203
7.3.2	Associative sorting algorithm	204
7.3.3	Hybrid sorting algorithms	209
7.4	Optimization problems	211
7.4.1	Linear programming problem	212
7.4.2	Mountain Climbing algorithm	216
7.4.3	The gradient method	217
7.4.4	Knapsack problem	222
7.5	Optimization graph-problems	223
7.5.1	The shortest path graph-problem	224
7.5.2	Problem of minimal skeleton for connected graph	227
7.5.3	Maximum flow problem	229
7.5.4	Matching and problem of minimal connectivity of a graph	233
8	Nontraditional computation procedures	238
8.1	Heuristics	238
8.2	Tree loping	239
8.3	Evolution algorithms	240
Annex		246
A.1	MST problem in historical context	246
A.2	MST problem in present terminology	252
A.2.1	Borůvka's solution	252
A.2.2	Jarník's solution	255
A.2.3	Kruskal's solution	258
A.3	Summary	259
A.4	Annex references	260
Bibliography		262
Index		267