Contents

Preface vii

List of symbols. Notations. Conventions xi

1 Introduction 1
- 1.1 Hardy's inequality . . . 1
- 1.2 Hardy's inequality with derivatives . . . 5
- 1.3 Some notations and modifications . . . 7
- 1.4 Hardy's inequality for some special classes of functions . . . 9
- 1.5 The role of the interval (a, b) . . . 11
- 1.6 Compactness of the Hardy operator . . . 15
- 1.7 Miscellanea . . . 20
- 1.8 Comments. Remarks. . . . 24

2 Some weighted norm inequalities 27
- 2.1 Preliminaries . . . 27
- 2.2 A special operator . . . 28
- 2.3 General Hardy-type operators. The fundamental lemma . . . 36
- 2.4 General Hardy-type operators. The case $p \leq q$ . . . 44
- 2.5 General Hardy-type operators. The case $p > q$ . . . 50
- 2.6 Some modifications and extensions . . . 59
- 2.7 Comments. Remarks. . . . 60

3 The Hardy-Steklov operator 63
- 3.1 Introduction . . . 63
- 3.2 Some auxiliary results . . . 68
- 3.3 The case $p \leq q$ . . . 70
- 3.4 The case $p > q$ . . . 74
- 3.5 Some applications . . . 81
- 3.6 Some generalizations and extensions . . . 86
- 3.7 Comments. Remarks. . . . 90

4 Higher order Hardy inequalities **93**
4.1 Preliminaries 93
4.2 Some special cases 95
4.3 The general case 99
4.4 Some special cases II 111
4.5 Reducing the conditions 115
4.6 Overdetermined classes ($k = 1$) 117
4.7 Overdetermined classes ($k > 1$) 122
4.8 Overdetermined classes (another approach) 132
4.9 Comments, remarks 143

5 Fractional order Hardy inequalities **145**
5.1 Introduction 145
5.2 An elementary approach — the unweighted case 147
5.3 The general weighted case 151
5.4 Hardy-type inequalities and interpolation theory 163
5.5 Further results 170
5.6 Comments. Remarks. 178

6 Integral operators on the cone of monotone functions **181**
6.1 Introduction 181
6.2 The duality principle of Sawyer 183
6.3 Applications of the duality principle 189
6.4 More general integral operators 194
6.5 Comments. Remarks. 200

References 203