



Abstract. The textbook *Ordinary Differential Equations* is a study utility for the study of the first-year course in mathematics for the students of the Faculty of Economics and Management at the University of Debrecen. It is focused on an exposition of the elementary theory of ordinary differential equations, especially to basic methods of finding solutions of ordinary differential equations.

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Introduction

Ordinary differential equations represents an important branch of mathematics. They belong to mathematical analysis, similarly as differential and integral calculus. Ordinary differential equations arise in many different contexts including geometry, mechanics, astronomy, biology, ecology, medicine, economy, especially within the scope of mathematical modelling. Many famous mathematicians have studied differential equations and contributed to the field, such as Newton, Leibniz, the Bernoullis, Cauchy, Riccati, Clairaut, d'Alembert, Euler, Lagrange, Laplace, Bessel, Legendre, Chebyshev and Ljapunov.

This textbook is written for economically and managerially specialized students. The important component of their erudition is mathematics and its applications in economy. The mathematical approach to economic analysis, so called mathematical economy, is indispensable for understanding of economic phenomena and has considerable and still increasing role in economic education. Therefore purely mathematical chapters on differential equations are supplemented by Chapter 3 and Chapter 5, where some simple economically oriented mathematical models utilizing ordinary differential equations are presented. The models are adapted from [2], [5], [6] and [16]. The readers, who are not interested in economic applications of differential equations can omit these two chapters.

The above mentioned mathematical chapters contain some common types of ordinary differential equations and the elementary methods of their solving. Naturally, with regard to the time limitations and the fact that supposed readers will not be specialists in mathematics, the list of the types of differential equations mentioned in the textbook is constrained and it covers by far not all the types of solvable differential equations which can be found in textbooks. For the same reason, we do not put the accent on the mathematical exactness and we confine ourselves only to unavoidable pieces of knowledge. For instance the systems of differential equations are not mentioned in the textbook at all. The proofs of statements are omitted. The used basic economic notions are not explained and the reader is supposed to be familiarized with them within the framework of courses in economics.

Notice finally that it is necessary to distinguish between mathematical economics and econometrics. Econometrics is concerned especially with measurement of economic data. It deals with the study of empirical observations using statistical methods of estimation and hypothesis testing. Mathematical economy, on the other hand, refers to the application of mathematics to various aspects of economic analysis, with little or no concern about the statistical problems as the errors of measurement of the variables under study. Nevertheless, empirical studies and theoretical analyses are often complementary and influencing each other. For details see [2].