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1.1 Basic Ideas of Multidimensional Scaling

The basic ideas of MDS are easily explained using a small example. Consider Table 1.1, it contains the correlations of different crimes in 50 US states. The correlations show, for example, that if there are many cases of Assault in a state, then there are also many cases of Murder ($r = 0.51$). We now use these correlations via MDS. This means that we try to represent the seven crimes by seven points in a geometric space so that any two points lie the closer together the greater the correlation of the crimes that these points represent.

To reach this goal, we take seven cards, and write the name of each crime on each of them, from Murder to Auto Theft. These cards are then arranged at random in an arbitrary arrangement (Fig. 1.1). Their distances are measured (Fig. 1.2) and compared with the correlations in Table 1.1. This comparison shows that the configuration in Fig. 1.1 does not represent the data in the desired sense. For example, the cards for Murder and Assault should be relatively close together, because these crimes are correlated with 0.51, whereas the cards for Murder and Larceny should be farther apart, as these crimes are correlated with only 0.06. We, therefore, try to move the cards repeatedly around – so-called “iteratively” – so that the distances correspond more closely to the data. Figure 1.3 demonstrates in which direction the cards should be shifted to improve the correspondence of data and distances.

Improving a given configuration iteratively by hand can be fairly tedious. It also does not guarantee convergence to a stable and optimal configuration. So, let us