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The genesis of this great and beautiful book spans more than 20 years. It collects and unifies many theoretical notions and results published by Bruno Courcelle and others in a large number of articles.

The concept of a language to communicate with a computer, a machine or any kind of device performing operations is at the heart of Computer Science, a field that has truly thrived with the emergence of symbolic programming languages in the 1960s. Formalizing the algorithms that enable computers to calculate an intended result, to control a machine or a robot, to search and find the relevant information in response to a query, and even to imitate the human brain in actions such as measuring risk and making decisions, is the main activity of computer scientists as well as of ordinary computer users.

The languages designed for these tasks, which number by thousands, are defined in the first place by syntactic rules that construct sets of words and to which are then attached meanings. This understanding of a language was first conceived by structural linguists, in particular Nicolaï Troubetzkoi, Roman Jakobson and Noam Chomsky, and has transformed Linguistics, the study of natural languages, by giving it new directions. It has also been extended to programming languages, which are artificial languages, and to the Lambda Calculus, one of many languages devised by logicians, among whom we can cite Kurt Gödel, Alonzo Church and Alan Turing, who aspired to standardize mathematical notation and to mechanize proofs. This same idea has inspired all research on computation theory and programming. Thanks to the results of this research, planes can fly with continuously monitored flight parameters, providing us with unprecedented reliability: this is so because millions of lines of code have been formally proved to be correct.

Words are strings of symbols taken from finite alphabets. They constitute the basic elements. They can represent all the information one might wish to capture, see, process, disseminate or share in a world that is fast becoming more and more "digital," as Gérard Berry emphasized recently in his lectures at the Collège de France.

Most information, though represented always by words, is nevertheless structured hierarchically and can thus be presented in a natural way as a tree or as a graph. Most