Contents

1 Two Central Questions of This Book and an Introduction to the Theories of Ill-posed and Coefficient Inverse Problems 1

1.1 Two Central Questions of This Book 2

1.1.1 Why the Above Two Questions Are the Central Ones for Computations of CIPs 4

1.1.2 Approximate Global Convergence 6

1.1.3 Some Notations and Definitions 11

1.2 Some Examples of Ill-posed Problems 14

1.3 The Foundational Theorem of A.N. Tikhonov 21

1.4 Classical Correctness and Conditional Correctness 23

1.5 Quasi-solution 25

1.6 Regularization 27

1.7 The Tikhonov Regularization Functional 31

1.7.1 The Tikhonov Functional 32

1.7.2 Regularized Solution 34

1.8 The Accuracy of the Regularized Solution for a Single Value of α 35

1.9 Global Convergence in Terms of Definition 1.1.2.4 39

1.9.1 The Local Strong Convexity 40

1.9.2 The Global Convergence 45

1.10 Uniqueness Theorems for Some Coefficient Inverse Problems 46

1.10.1 Introduction 46

1.10.2 Carleman Estimate for a Hyperbolic Operator 48

1.10.3 Estimating an Integral 56

1.10.4 Cauchy Problem with the Lateral Data for a Hyperbolic Inequality with Volterra-Like Integrals 57

1.10.5 Coefficient Inverse Problem for a Hyperbolic Equation ... 62

1.10.6 The First Coefficient Inverse Problem for a Parabolic Equation 68

1.10.7 The Second Coefficient Inverse Problem for a Parabolic Equation 70

1.10.8 The Third Coefficient Inverse Problem for a Parabolic Equation 76
1.10.9 A Coefficient Inverse Problem for an Elliptic Equation.... 78
1.11 Uniqueness for the Case of an Incident Plane Wave in Partial Finite Differences 79
1.11.1 Results 81
1.11.2 Proof of Theorem 1.11.1.1 83
1.11.3 The Carleman Estimate 85
1.11.4 Proof of Theorem 1.11.1.2 90

2 Approximately Globally Convergent Numerical Method 95
2.1 Statements of Forward and Inverse Problems 97
2.2 Parabolic Equation with Application in Medical Optics 98
2.3 The Transformation Procedure for the Hyperbolic Case 100
2.4 The Transformation Procedure for the Parabolic Case 103
2.5 The Layer Stripping with Respect to the Pseudo Frequency s 106
2.6 The Approximately Globally Convergent Algorithm 109
2.6.1 The First Version of the Algorithm 111
2.6.2 A Simplified Version of the Algorithm 112
2.7 Some Properties of the Laplace Transform of the Solution of the Cauchy Problem (2.1) and (2.2) 115
2.7.1 The Study of the Limit (2.12) 115
2.7.2 Some Additional Properties of the Solution of the Problem (2.11) and (2.12) 118
2.8 The First Approximate Global Convergence Theorem 122
2.8.1 Exact Solution 123
2.8.2 The First Approximate Global Convergence Theorem..... 125
2.8.3 Informal Discussion of Theorem 2.8.2 137
2.8.4 The First Approximate Mathematical Model 138
2.9 The Second Approximate Global Convergence Theorem 140
2.9.1 Estimates of the Tail Function 142
2.9.2 The Second Approximate Mathematical Model 151
2.9.3 Preliminaries 155
2.9.4 The Second Approximate Global Convergence Theorem.. 157
2.10 Summary 166

3 Numerical Implementation of the Approximately Globally Convergent Method 169
3.1 Numerical Study in 2D 170
3.1.1 The Forward Problem 171
3.1.2 Main Discrepancies Between the Theory and the Numerical Implementation 173
3.1.3 Results of the Reconstruction 174
3.2 Numerical Study in 3D 186
3.2.1 Computations of the Forward Problem 186

3.2.2 Result of the Reconstruction 188
3.3 Summary of Numerical Studies 191

4 The Adaptive Finite Element Technique and Its Synthesis with the Approximately Globally Convergent Numerical Method 193
4.1 Introduction 193
4.1.1 The Idea of the Two-Stage Numerical Procedure 193
4.1.2 The Concept of the Adaptivity for CIPs 194
4.2 Some Assumptions 196
4.3 State and Adjoint Problems 198
4.4 The Lagrangian 199
4.5 A Posteriori Error Estimate for the Lagrangian 202
4.6 Some Estimates of the Solution an Initial Boundary Value Problem for Hyperbolic Equation (4.9) 210
4.7 Fréchet Derivatives of Solutions of State and Adjoint Problems 216
4.8 The Fréchet Derivative of the Tikhonov Functional 222
4.9 Relaxation with Mesh Refinements 225
4.9.1 The Space of Finite Elements 226
4.9.2 Minimizers on Subspaces 229
4.9.3 Relaxation 233
4.10 From the Abstract Scheme to the Coefficient Inverse Problem 2.1 235
4.11 A Posteriori Error Estimates for the Regularized Coefficient and the Relaxation Property of Mesh Refinements 237
4.12 Mesh Refinement Recommendations 241
4.13 The Adaptive Algorithm 244
4.13.1 The Algorithm In Brief 244
4.13.2 The Algorithm 244
4.14 Numerical Studies of the Adaptivity Technique 245
4.14.1 Reconstruction of a Single Cube 246
4.14.2 Scanning Acoustic Microscope 248
4.15 Performance of the Two-Stage Numerical Procedure in 2D 258
4.15.1 Computations of the Forward Problem 258
4.15.2 The First Stage 261
4.15.3 The Second Stage 264
4.16 Performance of the Two-Stage Numerical Procedure in 3D 266
4.16.1 The First Stage 275
4.16.2 The Second Stage 277
4.17 Numerical Study of the Adaptive Approximately Globally Convergent Algorithm 279
4.17.1 Computations of the Forward Problem 285
4.17.2 Reconstruction by the Approximately Globally Convergent Algorithm 288
4.17.3 The Adaptive Part 289
4.18 Summary of Numerical Studies of Chapter 4 291

5 Blind Experimental Data 295
5.1 Introduction 295
5.2 The Mathematical Model 297
5.3 The Experimental Setup 298
5.4 Data Simulations 301
5.5 State and Adjoint Problems for Experimental Data 302
5.6 Data Pre-Processing 304
5.6.1 The First Stage of Data Immersing 304
5.6.2 The Second Stage of Data Immersing 307
5.7 Some Details of the Numerical Implementation of the Approximately... 309
5.7.1 Stopping Rule for 311
5.8 Reconstruction by the Approximately Globally Convergent Numerical Method 311
5.8.1 Dielectric Inclusions and Their Positions 311
5.8.2 Tables and Images 312
5.8.3 Accuracy of the Blind Imaging 314
5.8.4 Performance of a Modified Gradient Method 316
5.9 Performance of the Two-Stage Numerical Procedure 319
5.9.1 The First Stage 319
5.9.2 The Third Stage of Data Immersing 320
5.9.3 Some Details of the Numerical Implementation of the Adaptivity 323
5.9.4 Reconstruction Results for Cube Number 1 323
5.9.5 Reconstruction Results for the Cube Number 2 325
5.9.6 Sensitivity to the Parameters α and β 327
5.9.7 Additional Effort for Cube Number 1 327
5.10 Summary 332

6 Backscattering Data 335
6.1 Introduction 335
6.2 Forward and Inverse Problems 337
6.3 Laplace Transform 339
6.4 The Algorithm 340
6.4.1 Preliminaries 340
6.4.2 The Sequence of Elliptic Equations 342
6.4.3 The Iterative Process 344
6.4.4 The Quasi-Reversibility Method 345
6.5 Estimates for the QRM 346
6.6 The Third Approximate Mathematical Model 354
6.6.1 Exact Solution 354
6.6.2 The Third Approximate Mathematical Model 356
6.7 The Third Approximate Global Convergence Theorem 358

6.8 Numerical Studies 367
6.8.1 Main Discrepancies Between Convergence Analysis and Numerical Implementation 367
6.8.2 A Simplified Mathematical Model of Imaging of Plastic Land Mines 368
6.8.3 Some Details of the Numerical Implementation 369
6.8.4 Numerical Results 372
6.8.5 Backscattering Without the QRM 374
6.9 Blind Experimental Data Collected in the Field 376
6.9.1 Introduction 378
6.9.2 Data Collection and Imaging Goal 379
6.9.3 The Mathematical Model and the Approximately Globally Convergent Algorithm 381
6.9.4 Uncertainties 385
6.9.5 Data Pre-processing 388
6.9.6 Results of Blind Imaging 391
6.9.7 Summary of Blind Imaging 392

References 393

Index 401