Contents

1 A Tutorial Introduction to Python **1**

1.1 The IDLE Integrated Development Environment for Python 2

1.1.1 Tutorial One: Using Python as a Powerful Calculator 4

1.1.2 Tutorial Two: Simple Programming with Python . . . 6

1.1.3 Tutorial Three: Simple Plotting Using the Turtle Module 9

1.2 Anaconda, Spyder and the Libraries, Sympy, Numpy, and Matplotlib 14

1.2.1 Tutorial One: A Tutorial Introduction to Sympy . . . 15

1.2.2 Tutorial Two: A Tutorial Introduction to Numpy and Matplotlib 18

1.2.3 Tutorial Three: Simple Programming, Solving ODEs, and More Detailed Plots 20

1.3 Exercises 27

2 Differential Equations **33**

2.1 Simple Differential Equations and Applications 34

2.1.1 Linear Differential Equations 34

2.1.2 Separable Differential Equations 35

2.1.3 Exact Differential Equations 39

2.1.4 Homogeneous Differential Equations 40

2.2 Applications to Chemical Kinetics 44

2.3 Applications to Electric Circuits 48

2.4 Existence and Uniqueness Theorem 53

2.5 Python Programs 57

2.6 Exercises 59

3 Planar Systems **65**

3.1 Canonical Forms 66

3.1.1 Real Distinct Eigenvalues 68

3.1.2 Complex Eigenvalues ($\lambda = \alpha \pm i\beta$) 69

3.1.3 Repeated Real Eigenvalues 70

3.2 Eigenvectors Defining Stable and Unstable Manifolds 72
3.3 Phase Portraits of Linear Systems in the Plane 74
3.4 Linearization and Hartman's Theorem 78
3.5 Constructing Phase Plane Diagrams 79
3.6 Python Programs 87
3.7 Exercises 90

4 Interacting Species **95**
4.1 Competing Species 96
4.2 Predator-Prey Models 99
4.3 Other Characteristics Affecting Interacting Species 104
4.4 Python Programs 107
4.5 Exercises 108

5 Limit Cycles **113**
5.1 Historical Background 114
5.2 Existence and Uniqueness of Limit Cycles in the Plane . . . 117
5.3 Nonexistence of Limit Cycles in the Plane 123
5.4 Perturbation Methods 127
5.5 Python Programs 136
5.6 Exercises 139

6 Hamiltonian Systems, Lyapunov Functions, and Stability **145**
6.1 Hamiltonian Systems in the Plane 146
6.2 Lyapunov Functions and Stability 151
6.3 Python Programs 157
6.4 Exercises 158

7 Bifurcation Theory **163**
7.1 Bifurcations of Nonlinear Systems in the Plane 164
7.1.1 A Saddle-Node Bifurcation 165
7.1.2 A Transcritical Bifurcation 167
7.1.3 A Pitchfork Bifurcation 167
7.1.4 A Hopf Bifurcation 170
7.2 Normal Forms 170
7.3 Multistability and Bistability 174
7.4 Python Programs 178
7.5 Exercises 180

8 Three-Dimensional Autonomous Systems and Chaos **185**
8.1 Linear Systems and Canonical Forms 186
8.2 Nonlinear Systems and Stability 190
8.3 The Rössler System and Chaos 194

8.3.1 The Rössler Attractor 194
8.3.2 Chaos 196
8.4 The Lorenz Equations, Chua's Circuit, and the Belousov-Zhabotinski Reaction 199
8.4.1 The Lorenz Equations 199
8.4.2 Chua's Circuit 201
8.4.3 The Belousov-Zhabotinski (BZ) Reaction 204
8.5 Python Programs 207
8.6 Exercises 211

9 Poincaré Maps and Nonautonomous Systems in the Plane 215
9.1 Poincaré Maps 216
9.2 Hamiltonian Systems with Two Degrees of Freedom 221
9.3 Nonautonomous Systems in the Plane 227
9.4 Python Programs 235
9.5 Exercises 239

10 Local and Global Bifurcations 245
10.1 Small-Amplitude Limit Cycle Bifurcations 246
10.2 Gröbner Bases 252
10.3 Melnikov Integrals and Bifurcating Limit Cycles from a Center 258
10.4 Bifurcations Involving Homoclinic Loops 260
10.5 Python Programs 262
10.6 Exercises 266

11 The Second Part of Hilbert's Sixteenth Problem 271
11.1 Statement of Problem and Main Results 272
11.2 Poincaré Compactification 275
11.3 Global Results for Liénard Systems 281
11.4 Local Results for Liénard Systems 290
11.5 Python Programs 291
11.6 Exercises 292

12 Delay Differential Equations 297
12.1 Introduction and the Method of Steps 298
12.2 Applications in Biology 304
12.3 Applications in Nonlinear Optics 310
12.4 Other Applications 313
12.5 Python Programs 315
12.6 Exercises 320

13 Linear Discrete Dynamical Systems **327**
13.1 Recurrence Relations 328
13.2 The Leslie Model 333
13.3 Harvesting and Culling Policies 337
13.4 Python Programs 342
13.5 Exercises 343

14 Nonlinear Discrete Dynamical Systems **347**
14.1 The Tent Map and Graphical Iterations 348
14.2 Fixed Points and Periodic Orbits 353
14.3 The Logistic Map, Bifurcation Diagram, and Feigenbaum Number 360
14.4 Gaussian and Hénon Maps 368
14.5 Applications 373
14.6 Python Programs 376
14.7 Exercises 380

15 Complex Iterative Maps **385**
15.1 Julia Sets and the Mandelbrot Set 386
15.2 Boundaries of Periodic Orbits 391
15.3 The Newton Fractal 395
15.4 Python Programs 396
15.5 Exercises 399

16 Electromagnetic Waves and Optical Resonators **403**
16.1 Maxwell's Equations and Electromagnetic Waves 404
16.2 Historical Background 406
16.3 The Nonlinear SFR Resonator 412
16.4 Chaotic Attractors and Bistability 413
16.5 Linear Stability Analysis 417
16.6 Instabilities and Bistability 420
16.7 Python Programs 424
16.8 Exercises 428

17 Fractals and Multifractals **433**
17.1 Construction of Simple Examples 434
17.2 Calculating Fractal Dimensions 441
17.3 A Multifractal Formalism 448
17.4 Multifractals in the Real World and Some Simple Examples 452
17.5 Python Programs 459
17.6 Exercises 464

18 Image Processing with Python **471**
18.1 Image Processing and Matrices 472
18.2 The Fast Fourier Transform 477
18.3 The Fast Fourier Transform on Images 484
18.4 Exercises 487

19 Chaos Control and Synchronization **491**
19.1 Historical Background 492
19.2 Controlling Chaos in the Logistic Map 497
19.3 Controlling Chaos in the Hénon Map 498
19.4 Chaos Synchronization 505
19.5 Python Programs 509
19.6 Exercises 513

20 Neural Networks **519**
20.1 Introduction 520
20.2 The Delta Learning Rule and Backpropagation 526
20.3 The Hopfield Network and Lyapunov Stability 531
20.4 Neurodynamics 541
20.5 Python Programs 545
20.6 Exercises 550

21 Binary Oscillator Computing **557**
21.1 Brain Inspired Computing 558
21.2 Oscillatory Threshold Logic 563
21.3 Applications and Future Work 571
21.4 An Assay for Neuronal Degradation 577
21.5 Python Programs 579
21.6 Exercises 584

22 Coursework and Examination-Type Questions **591**
22.1 Examples of Coursework Questions 592
22.2 Examination 1 607
22.3 Examination 2 610
22.4 Examination 3 613

23 Solutions to Exercises **619**
23.1 Chapter 1 619
23.2 Chapter 2 622
23.3 Chapter 3 623
23.4 Chapter 4 625
23.5 Chapter 5 626
23.6 Chapter 6 628
23.7 Chapter 7 628

23.8 Chapter 8 630
23.9 Chapter 9 631
23.10 Chapter 10 631
23.11 Chapter 11 632
23.12 Chapter 12 634
23.13 Chapter 13 634
23.14 Chapter 14 636
23.15 Chapter 15 637
23.16 Chapter 16 638
23.17 Chapter 17 638
23.18 Chapter 18 639
23.19 Chapter 19 639
23.20 Chapter 20 640
23.21 Chapter 21 640
23.22 Chapter 22 641

Appendix A Index of Python Programs **645**
A.1 IDLE Python Programs 645
A.2 Anaconda Python Programs 646

Index **651**