

101 References

101 Index

Part I Index

Contents

Impossibility, Proofs of Nonexistence

Introduction v

Part I. Proofs of Impossibility, Proofs of Nonexistence 1

1. Proofs of Irrationality 3

2. The Elements of the Theory of Geometric Constructions 11

3. Constructible Regular Polygons 17

4. Some Basic Facts About Linear Spaces and Fields 21

5. Algebraic and Transcendental Numbers 27

6. Cauchy's Functional Equation 33

7. Geometric Decompositions 39

Part II. Constructions, Proofs of Existence 47

8. The Pigeonhole Principle 49

9. Liouville Numbers 55

10. Countable and Uncountable Sets 59

11. Isometries of \mathbf{R}^n 67

12. The Problem of Invariant Measures 75

13. The Banach–Tarski Paradox 81

14. Open and Closed Sets in \mathbf{R} . The Cantor Set 85

15. The Peano Curve 93

16. Borel Sets 97

17. The Diagonal Method 103

References 107
 Hints 109
 Index 117

Contents

Introduction v

Part I. Proofs of Impossibility Proofs of Nonexistence

1. Proofs of Irrationality 3
 2. The Elements of the Theory of Geometric Constructions 11
 3. Constructible Regular Polygons 17
 4. Some Basic Facts About Linear Spaces and Fields 21
 5. Algebraic and Transcendental Numbers 27
 6. Cauchy's Functional Equation 33
 7. Geometric Decompositions 39

Part II. Constructive Proofs of Existence

8. The Pigeonhole Principle 43
 9. Liouville Numbers 49
 10. Countable and Uncountable Sets 59
 11. Isometries of \mathbb{R}^n 67
 12. The Problem of Invariant Measures 75
 13. The Banach-Tarski Paradox 81
 14. Open and Closed Sets in \mathbb{R} . The Cantor Set 85
 15. The Peano Curve 93
 16. Borel Sets 97
 17. The Diagonal Method 103