

# CONTENTS

PREFACE . . . . .	IX
CHAPTER I. INTRODUCTION . . . . .	1
1. The motivations . . . . .	1
2. An informal introduction to formalized languages . . . . .	5
3. Assigning meanings to programs . . . . .	11
4. Semantic properties of programs . . . . .	16
5. Expressivity. An introduction to the language of algorithmic logic . . . . .	18
6. On applications . . . . .	20
CHAPTER II. LOGIC OF DETERMINISTIC ITERATIVE PROGRAMS . . . . .	23
1. Language . . . . .	24
2. Semantics . . . . .	30
3. Expressiveness . . . . .	38
4. Properties of the semantic consequence operation . . . . .	51
5. Axiomatization . . . . .	56
6. Models and consistency . . . . .	65
7. Useful tautologies and inference rules . . . . .	69
8. An example of a correctness proof . . . . .	75
Bibliographic remarks . . . . .	77
CHAPTER III. METAMATHEMATICAL INVESTIGATIONS OF ALGORITHMIC LOGIC . . . . .	79
1. Lindenbaum algebra . . . . .	79
2. The Completeness Theorem . . . . .	89
3. Two corollaries of the Completeness Theorem . . . . .	95
4. The standard execution method is implicitly defined by the axiomatization of algorithmic logic . . . . .	97

5. Gentzen type axiomatization . . . . .	103
6. The normal form of programs . . . . .	109
7. Equality . . . . .	115
8. Generalized terms . . . . .	119
9. Partial functions . . . . .	122
10. Many sorted structures . . . . .	127
11. Definability and programmability . . . . .	131
12. Inessentiality of definitions . . . . .	135
Bibliographic remarks . . . . .	137

#### CHAPTER IV. ALGORITHMIC PROPERTIES OF DATA

STRUCTURES . . . . .	138
1. Data structures in programming . . . . .	138
2. Dictionaries . . . . .	141
3. Theory of dictionaries . . . . .	142
4. Representation theorem for models of ATD . . . . .	149
5. On complexity of ATD . . . . .	151
6. The theory of priority queues . . . . .	154
7. The theory of natural numbers . . . . .	155
8. Stacks . . . . .	159
9. The theory of stacks . . . . .	160
10. The representation theorem for stacks . . . . .	164
11. Implementation of arithmetic and dictionaries . . . . .	166
12. Theory of links and stacks—ATSL . . . . .	167
13. Implementation of stacks in LOGLAN programming language . . . . .	173
14. Queues . . . . .	176
15. Binary trees . . . . .	179
16. Binary search trees . . . . .	181
17. An interpretation of the theory of priority queues . . . . .	184
18. An implementation of priority queues . . . . .	187
19. Arrays . . . . .	190
20. Hashtables . . . . .	193
21. Rational numbers . . . . .	194
22. Complex numbers . . . . .	195
23. Real numbers . . . . .	200
24. Concluding remarks . . . . .	202
Bibliographic remarks . . . . .	204

CHAPTER V. PROPOSITIONAL ALGORITHMIC LOGIC	206
1. Syntax and semantics . . . . .	208
2. Semantic properties of program schemes . . . . .	212
3. Properties of semantic structures . . . . .	221
4. The semantic consequence operation is not compact . . . . .	228
5. The syntactic consequence operation . . . . .	229
6. Examples of propositional theories . . . . .	233
7. Lindenbaum algebra . . . . .	237
8. Deterministic and total interpretations of atomic programs	239
9. Partial functional interpretations . . . . .	243
10. Bounded non-determinism: The Completeness Theorem .	248
11. Elimination of bounded non-deterministic program variables . . . . .	257
12. Yanov schemes . . . . .	261
13. Application of PAL in microprogramming . . . . .	263
Bibliographic remarks . . . . .	268
 CHAPTER VI. NON-DETERMINISM IN ALGORITHMIC LOGIC . . . . .	 269
1. Non-deterministic <b>while</b> -programs and their semantics . . . . .	270
2. Properties of non-deterministic programs . . . . .	273
3. The Substitution Theorem . . . . .	277
4. Non-deterministic algorithmic logic . . . . .	282
5. Certain metamathematical results . . . . .	286
6. On isomorphism of data structures . . . . .	289
7. On the equivalence of non-deterministic programs . . . . .	291
Bibliographic remarks . . . . .	297
 CHAPTER VII. PROBLEMS AND THEORIES INSPIRED BY THE LOGLAN PROJECT . . . . .	 298
1. Concurrent programs . . . . .	299
2. MAX semantics . . . . .	300
3. Comparison with some other concepts of concurrency . . . . .	303
4. A comparison of MAX and ARB semantics in the case of Petri nets . . . . .	311
5. Critical remarks concerning MAX semantics . . . . .	315
6. LIBERAL semantics . . . . .	318

7. An algorithmic theory of references . . . . .	328
8. Representation theorem for ATR theory . . . . .	332
9. Specification of univocal references . . . . .	338
10. Virtual memory . . . . .	339
11. Concatenable type declarations . . . . .	341
12. An implementation of rational numbers . . . . .	344
Bibliographic remarks . . . . .	346
APPENDIX A. BOOLEAN ALGEBRAS . . . . .	348
APPENDIX B. THE PROOF OF LEMMA 2.2 FROM CHAPTER III . . . . .	352
BIBLIOGRAPHY . . . . .	356
INDEX . . . . .	369