

# Contents

PREFACE		v
CHAPTER 1.	PRELIMINARIES	1
	1. Notation and terminology	1
	2. Polynomial algebras	7
	3. Integral extensions	29
	4. Tensor products	32
	5. Module-theoretic prerequisites	37
	6. Topological prerequisites	41
CHAPTER 2.	CLASSICAL TOPICS IN FIELD THEORY	47
	1. Algebraic extensions	47
	2. Normal extensions	57
	3. Separable, purely inseparable and simple extensions	63
	4. Galois extensions	79
	5. Finite fields, roots of unity and cyclotomic extensions	85
	6. Norms, traces and their applications	103
	7. Discriminants and integral bases	117
	8. Units in quadratic fields	135
	9. Units in pure cubic fields	148
	10. Finite Galois theory	167
	11. Profinite groups	172
	12. Infinite Galois theory	184
	13. Witt vectors	192
	14. Cyclic extensions	205
	15. Kummer theory	214
	16. Radical extensions and related results	221
	17. Degrees of sums in a separable field extension	242
	18. Galois cohomology	247
	19. The Brauer group of a field	263
	20. An interpretation of $H_0^3(G, E^*)$	282
	21. A cogalois theory for radical extensions	303
	22. Abelian $p$ -extensions over fields of characteristic $p$	323
	23. Formally real fields	333
	24. Transcendental extensions	348
CHAPTER 3.	VALUATION THEORY	353
	1. Valuations	353
	2. Valuation rings and places	368
	3. Dedekind domains	376
	4. Completion of a field	389
	5. Extensions of valuations	400
	6. Valuations of algebraic number fields	411

7.	Ramification index and residue degree	414
8.	Structure of complete discrete valued fields	421
	A. Notation and terminology	421
	B. The equal characteristic case	422
	C. The unequal characteristic case	427
	D. The inertia field	431
	E. Cyclotomic extensions of $p$ -adic fields	436
<b>CHAPTER 4.</b>	<b>MULTIPLICATIVE GROUPS OF FIELDS</b>	<b>439</b>
	1. Some general observations	439
	2. Infinite abelian groups	443
	3. The Dirichlet-Chevalley-Hasse Unit Theorem	449
	4. The torsion subgroup	461
	5. Global fields	463
	6. Algebraically closed, real closed and the rational $p$ -adic fields	468
	7. Local fields	474
	A. Preparatory results	475
	B. The equal characteristic case	481
	C. The unequal characteristic case	482
	8. Extensions of algebraic number fields	487
	9. Brandis's theorem	496
	10. Fields with free multiplicative groups modulo torsion	501
	11. A nonsplitting example	517
	12. Embedding groups	519
	13. Multiplicative groups under field extensions	525
	14. Notes	531
<b>BIBLIOGRAPHY</b>		<b>535</b>
<b>NOTATION</b>		<b>541</b>
<b>INDEX</b>		<b>547</b>