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### State of the Art: In Search of a Fully Microscopic Theory of Glasses

Most classical solid state textbooks are almost entirely devoted to crystals, see, e.g., [21]. The main reason is that, while the theory of crystalline solids is fully developed, that of amorphous solids is still very incomplete. As a first approximation, crystals can be understood as perfectly symmetric periodic lattices, around which particles undergo small vibrations. A low-temperature harmonic expansion can then be constructed to express the thermodynamic properties in terms of harmonic excitations – i.e., phonons – and anharmonic (quasiparticles) can then be added to the theory to describe crystal thermal expansion and melting [180, 212]. Crystals are well understood mainly because their structure is well understood as small perturbations of a perfectly symmetric lattice, the small parameter being the amplitude of thermal vibrations and the density of defects.

Yet most of the solid matter in nature is not crystalline. Foams, pastes, granulars and plastics are but a few examples. Glasses are not only ubiquitous but also extremely important for practical reasons. Glasses display all kind of anomalies with respect to crystals: in particular, their vibrations cannot simply be understood in terms of plane waves, their flow is not mediated by well defined defects, and their dynamics is extremely complex. Unlike crystals, glasses offer no guiding symmetry principle to construct a microscopic theory, and no natural ‘small’ parameter can be used to organise a perturbative expansion.

Constructing a complete first-principle theory of glasses has then turned out to be an extremely difficult task. Yet a lot of progress has been made and, recently, several books on glasses written (or edited) by theoretical physicists [37, 53, 168, 357] have appeared. These books are largely devoted to the phenomenology of real (or realistic models of) materials, as known from experiments and numerical simulations, and the theoretical approaches they discuss mostly make use of approximate